A DISCRIMINANT ANALYSIS OF THE BANKRUPTCY OF SECURITIES BROKERAGE/DEALER FIRMS

By

Iraj Afkham

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To my family:

Maryam, my wife, and
Ladan and Mahnoosh, my daughters

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By

Iraj Afkham

December, 1975

Chairman: Henry R. Fogler
Co-Chairman: Robert C. Radcliffe
Major Department: Management

This dissertation is an attempt to meet the following objectives: (1) to identify potential economic and non-economic factors which might differentiate between financially distressed firms and those which appear to be economically viable; (2) to statistically determine a set of variables which has the highest discriminatory power; and (3) to suggest both public and managerial action which might be taken in light of the statistical results obtained in point 2.

To begin the analysis, it was hypothesized that the probability of financial distress of a firm at time t is a function of the net cash-flow per dollar of assets at time t, the extent of the risk exposure, and the mismanagement of the firm.

Three potential approaches for predicting failure are discussed in the dissertation: "deterministic," probabilistic, and blend approach. The "deterministic" approach was meant to define the traditional method which uses the financial ratios in conjunction with the MDA technique. The probabilistic approach was stated to be the one which does not treat the financial ratios and other related forces as factors whose values can be determined with certainty at any point of time in the near future. Rather, it attaches some probability distribution to each factor, based upon its historical performance. And, instead of using the MDA technique, it relies upon the modern probability theories.

The probabilistic approach suggested in this study, although intriguing, requires a great deal of research before it can be put to work. The blend approach, used in the present study, was therefore proposed to act as a bridge between the traditional and the probabilistic approaches. It retains most of the favorable properties of the traditional approach, specially with respect to the MDA technique, yet incorporates some facets of the probabilistic approach.

After presenting an analytic framework for utilization of the blend approach and providing a groundwork for the application of the MDA technique in two successive chapters, an effort was made to empirically test the hypothesis. Due to the confidentiality of a part of financial data, however, the hypothesis had to be modified by substituting the cash-flow element with profitability.

Sixty SIPC failed and 60 nonfailed firms were selected and the original and holdout samples were formed by halving the entire observations and assigning each half randomly to one sample. The main thrust of the empirical part rests upon Test I whose predictive span is about one year. An attempt was also made to extend the analysis to two years prior to bankruptcy, in Test II.

Test I examines 83 variables and selects 12 of them as an "optimal" subset by a multi-stage stepwise selection procedure. The discriminant model developed in this test successfully differentiates between failed and nonfailed groups. The quadratic classification procedure correctly classifies 98.33 percent of the original sample and misclassifies, on average, 13.33 percent of the holdout sample. These results, along with the results of applying the scrambled method, Lachenbruch classification procedure, and "normalized" model, reveal the relatively high accuracy and validity of the model.

The 12 selected variables measure profitability, risk, and mismanagement of a firm. The relative discriminatory powers of these variables in the model reveal that the risk element is the most significant determinant of the firm's survival. Profitability and mismanagement are in the second and third places. This may help managements in making decisions when considering risky ventures.

Contrary to Test I, the result of Test II was expectedly less than successful. This was partly due to the nature of the securities business and partly due to the type of data available.

CHAPTER I

INTRODUCTION

The Backbone of the Capital Market

During 1974, the market value of all security transactions on the United States registered exchanges amounted to $124.7 billion. The significance of the figure may become more apparent if it is considered in conjunction with the nation's Gross National Product in that year. The GNP in 1974 amounted to $1,397.4 billion and the value of the securities transactions was as large as an amount equal to about 9 percent of the GNP. Although this is the lowest percentage in the past few years (the highest is 23, in 1968), it strikingly reveals the important role that the securities broker/dealers play in flowing funds in the nation's capital market.

The huge volume of securities transactions in each year is handled by several thousands of broker/dealer firms. In 1973, for example, the number of registered broker/dealer firms was 4,088. These firms had employed a total of about 349,000 employees to channel the savings of individuals to corporate users and to transfer the ownership of outstanding securities from one investor to another. The total assets marshalled by the securities firms having gross securities income of $20,000 and over amounted to $25.9 billion.

In order to present a brief profile of the securities broker/dealer firms, similar statistics as those stated above are tabulated in Table 1 for the period 1969-1974. This may aid in understanding the general position of the securities industry in the overall economy.

Literature Review

The maintenance of fair and orderly markets for securities has long been one of the highly desired objectives of the legislatures, administrators, courts, and various self-regulatory organizations. The Securities Act of 1933 and 1934 along with the creation of the Securities and Exchange Commission (SEC) to administer the acts, as well as the pursuant amendment acts of 1939, 1940, 1964, 1970, 1974, and 1975 partially reflect the main concern of these authorities in providing conditions designed to create securities markets capable of meeting the nation's need for investment capital while operating in the public interest and serving to protect the public investor.

For a long time, the going presumption was that the efforts directed toward preventing fraudulent, deceptive and manipulative acts in the securities markets would protect the individual investors and the public interest. It was not until 1970 that the hardship and sometimes disaster which were faced by the individual investors whose broker/dealers went bankrupt or underwent liquidation were widely

TABLE 1. SOME SELECTED STATISTICS WHICH PROVIDE A BRIEF PROFILE OF THE SECURITIES BROKER/DEALER FIRMS FOR THE YEARS 1969-74. (WHERE NOT SPECIFIED, DOLLARS ARE IN MILLIONS.)

			No. of Firms					
(1) Year	(2) Volume of Securities Transactions, $	(3) GNP (in Billions, Current $)	(4) Total	(5) Firms with $20,000 or More Income	(6) No. of Employees (in Thousands)	(7)* Total Assets	(8)* Operating Income (Before Taxes)	(9)* Return on Assets (Before Taxes) (8) ÷ (7)
1969	179,798	725.6	4,681	2,619	366	24,213	561	.023
1970	135,889	722.5	4,614	2,332	338	24,442	271	.011
1971	193,821	745.4	4,553	2,539	353	28,705	1,279	.044
1972	213,548	790.7	4,255	2,512	386	33,728	976	.029
1973	186,173	839.2	4,088	2,164	349	25,948	57	.002
1974	124,709	821.2	3,982	NA	NA	NA	NA	NA

*The figures in this column are related to the firms having gross securities income of $20,000 and over, as reported in column (5).

NA indicates data not available.

Note: Figures in column 2 are for the U.S. registered exchanges only.

Source: For columns (2) and (3), 1974 Statistical Abstracts of the United States.[1]
For columns (4), (5), (6), (7), and (8), 1974 Securities and Exchange Commission Annual Reports.[2]

realized. The substantial operational and financial problems that the securities industry experienced during the period from 1967 through 1970 had significantly diminished the investors' confidence in the markets; and the need for urgent corrective action was felt everywhere.

In December 1970, Congress passed the Securities Investor Protection Act to restore public confidence in the safety of the securities markets. This legislation established the Securities Investor Protection Corporation (SIPC). And, the corporation provided a limited insurance for the customer accounts of the securities industry.

During its four years of existence, SIPC has placed a total of 109 failed firms in liquidation. It has received a total of 97,230 customer claims. And, as of December 31, 1974, it has distributed securities and cash with an aggregate value of $241 million to satisfy these claims. Of this amount, approximately $202 million was derived from debtor estates. The remaining $39 million, however, was provided by SIPC through its advances to the trustees. SIPC's assessments on member firms during the four-year period totaled $106 million.[3]

To minimize the exposure of the SIPC fund, which is backed by a billion dollars of taxpayers' monies, Congress directed the SEC, at the time of passing the Securities Investor Protection Act, to compile a list of unsafe and unsound practices by broker/dealers and to report on the corrective steps being taken. Before the SEC report is reviewed, the results of another study which was published prior to the completion of this report is presented.

Review of SEC Records of the Demise of Selected Broker/Dealers

This is the title of a staff study undertaken for the Special Subcommittee on Investigations of the Committee on Interstate and Foreign Commerce of the House of Representatives.[4] The need for this study was stimulated by the crisis of the securities industry over the period 1967-70 which had brought disaster to numerous broker/dealers and financial hardship for the public investors. The study is a review of the files of the commission's headquarters offices for 46 selected broker/dealers who went into some form of liquidation during the 3-year period ending December 31, 1970. It also incorporates additional information obtained from other sources; and includes case studies for all 46 firms.

The review concluded, among other things, that there were many reasons for the failures of the subject firms and the factors were both internal and external to the firms. Internal factors included capital deficiency, operational losses, mismanagement and inefficiency in operating the firms' business, over-expansion of operations, inoperable and overly expensive modernization programs, back-office difficulties, irresponsible actions by principals, and

and fraud. External factors included a reduced volume of trading and a large decline in prices of many stocks during the period.

Study of Unsafe and Unsound Practices of Brokers and Dealers[5]

This study was undertaken by the regular staff of the SEC pursuant to Section 11(h) of the SIPC Act. Like the previous study, this one is also on the financial and operational crisis of 1967-70, but on a larger scale and in a more comprehensive way. It examines industry conditions and practices which permitted the crisis to develop. To name some, but not all, the report cites the following unsound practices:

- Inadequacy and inpermanence of capital.
- Over-emphasis on sales and trading activities at the expense of operational resources.
- Absence of control of securities traffic.
- Absence of an effective early warning system in self-regulatory organizations and their inability to respond to the crisis with meaningful corrective measures.
- Lack of experience of principal members of many, mainly small, firms.

The study reviews corrective measures already instituted, and suggests further actions to be taken by the broker/dealer firms, self-regulatory organizations, and the

SEC. The corrective measures detailed in the report include, among other things, increasing capital requirements, revising or proposing rules to strengthen control over securities and to provide greater protection for customers' free credit balances and securities left with brokers, and taking measures to provide the commission and self-regulatory authorities with more effective early warning systems.

Securities Industry Study by the Subcommittee on Securities[6]

During consideration of the SIPC legislation, the Committee on Banking, Housing, and Urban Affairs of the Senate concluded that a complete study of the securities industry and the securities markets should be undertaken by its Subcommittee on Securities. And, after authorization by the Senate in June 1971, the study was launched.

This study is the culmination of the analysis of the information obtained from a variety of sources. Some of the information was obtained directly from industry and government sources including responses to a detailed questionnaire on institutional membership sent to 104 firms, organizations and individuals. Other information was obtained through holding 18 days of hearings and receiving testimony from 78 witnesses. It includes also four detailed case studies of the operation of the regulatory system in the securities industry.

The contents of the study give the impression that the sub-committee tends to rely more on the market mechanism and its competitive nature than on regulations. It also looks beyond the current problems to the long-range structural features of the industry, markets, and regulatory system.

The basic conclusions of the study include, but are not limited to, the adaptation of greater flexibility in the industry, the redefinition of the division of responsibility between SEC and self-regulatory organizations, the provision of fair, orderly, and open procedure in reaching important decisions by these organizations, and giving more weight to competition than to regulation. In the words of the sub-committee report:

> ...Regulation, by government and by industry groups, is an essential element in protection of investors, but is not an effective substitute for competition in assuring a flexible and healthy industry.
>
> The concept of industry self-regulation, subject to SEC oversight, is well-adapted to dealing with problems of conduct and ethics, but it is not well-adapted to dealing with general economic questions involving competitive interrelationships among firms within the industry....[7]

Securities Industry Study by the Subcommittee on Commerce and Finance[8]

This study was undertaken in fulfillment of the commitment made by the chairman of the committee on Interstate and Foreign Commerce with respect to the committee's intention to make a complete study of the securities market and exchanges.

The study is largely the outgrowth of an extensive number of panel hearings, witness testimonies, views of industry leaders, interviews in virtually every section of the country, and reviews of previous and current studies on the securities industry.

It includes chapters on what went wrong in the securities business during the period 1967-70, criminal enforcement of securities laws, standards for entry into the securities business, capital for broker/dealers, protection of customers, accounting and auditing procedures, operational difficulties, cooperative regulations, central market system, commission rates, membership qualification for exchanges, antitrust aspects, and the SEC budget.

Space does not allow the enumeration of the large number and variety of conclusions reached, but to provide a feeling about the nature of this study, a sentence from the related letter of transmittal signed by J.E. Moss, Chairman of the subcommittee, is quoted here.

> ...This study is, to my knowledge, the only comprehensive congressional examination of the securities industry to have been completed within the time span of one Congress....

SIPC Study[9]

SIPC has also made inquiries of the trustees and reviewed the reports of the staffs of the self-regulatory organizations participating in the cases of the 94 broker/dealer firms liquidated as of December 31, 1973, in an effort to determine the cause of failure.

The study indicates that the matter of possible fraud and manipulation which has surfaced in many of these liquidations must be recognized as a major factor in these failures. Large concentrations of speculative issues in the trading activities contributed to other failures. Inadequate, inaccurate, and sometimes nonexistent books and records were another significant factor. In many cases, the operating management did not have the qualifications or experience needed to operate a general securities business. Lack of adequate capital continues to be mentioned by the trustees as a major factor in firm failures. According to the study, this condition results from a number of factors including too small a capital base, temporary illiquidity, over-commitment in particular issues, inability to absorb an adverse market movement, too rapid expansion, improper controls, and operating losses due to reduced volume. The study states that there was over-reliance on subordinated capital in a number of instances. In other cases, the subordination agreement was allegedly improperly executed, fraudulently induced or improperly withdrawn placing the firm in net capital violation.

The Early Warning Requirement of the SIPC Act[10]

Section 5(a)(1) of the SIPC Act requires that the commission or the self-regulatory organizations notify SIPC immediately upon discovery of facts which indicate that a broker or dealer subject to their regulations is in or approaching financial difficulty.

After receiving such notification, SIPC determines whether to apply for a court decree to initiate liquidation or to defer action until further information is obtained and doubt is removed.

There are five conditions specified in Section 5(h) of the Act. At least one of these conditions must be found to exist by SIPC to apply for a court decree and by the court to issue the decree in every case. The court shall issue a decree if it finds that the member firm:

a. Is insolvent within the meaning of Section 1(19) of the Bankruptcy Act, or is unable to meet its obligations as they mature, or

b. Has committed an act of bankruptcy within the meaning of Section 3 of the Bankruptcy Act, or

c. Is the subject of a proceeding pending in any court or before any agency of the United States or any state in which a receiver, trustee, or liquidator for such member has been appointed, or

d. Is not in compliance with applicable requirements under the 1934 Acts or rules or regulations of the commission or any self-regulatory organizations with respect to financial responsibility or hypothecation of customers' securities, or

e. Is unable to make such computations as may be necessary to establish complaince with such financial responsibility or hypothecation rules or regulations.

As was mentioned above, the SIPC Act requires that, when it appears to the SEC or any self-regulatory organization that a broker-dealer is in or approaching financial difficulty, SIPC is to be notified immediately. The statute, however, does not define "financial difficulty." Neither has it seemed practicable for SIPC to attempt to define these terms. In view of the number of self-regulatory organizations involved and the differences in their rules, procedures, problems, and the manner in which the various reporting and surveillance systems operate, SIPC has considered it appropriate, up to this point at least, to work with each organization separately.

The regulators fulfill the Act's requirement through regulatory procedures which integrate inspection and reporting programs with an early warning procedure for notifying SIPC. The primary objective of these programs is the early identification of those members which are in or approaching financial or operational difficulty and the initiation of action necessary to protect customers.

The Existing Early Warning Systems

All over the entire securities industry emphasis is now on the early warning systems by which the financial difficulty of the broker/dealers is to be detected in advance. The two main purposes of such early warning systems are: (1) to afford a self-regulatory organization an opportunity to initiate procedures so that the broker/dealer in question does not become a SPIC casualty, and (2) to provide a basis for notification to SIPC. Inasmuch as these warning systems are directly related to the subject matter of the present study, further review of the related material seems appropriate.

The early warning system of the SEC.[11] The SEC, in discharging its duties, periodically reviews through on-site inspections and in-house studies the early warning surveillance tools of the self-regulatory organizations to ensure that they constitute sound and effective programs.

The commission's early warning and surveillance tools include Rule 17a-11, Rule 17a-5(j), and Rule 17a-10. Rule 17a-11 was adopted to provide the commission and the self-regulatory authority with an adequate and timely flow of information on the financial and operational condition of broker/dealers. Among other provisions, it requires a broker/dealer to notify, by telegram, the commission and the appropriate self-regulatory organization if it breaks through certain specified financial or operational parameters. Rule 17a-5(j) requires a broker/dealer to notify the commission if its exchange membership is terminated and becomes subject to the commission's net capital rule. Rule 17a-10 requires a broker/dealer to file Form X-17A-10 annually with the commission.

An additional financial responsibility rule, Rule 15c3-3, titled "Customer Protection--Reserves and Custody of Securities," became effective on January 15, 1973. Failure to comply with the requirements of this rule has, in a number of instances, been the basis of the statutory condition.

In addition to these rules, the commission, working together with the NASD and the self-regulatory authorities, has developed a monthly, combined early warning list and a Rule 17a-11 list for each regional office. Each month the self-regulatory organizations submit, for the combined list, data pertaining to the financial condition of troubled firms within each commission region. The list is then speedily transmitted to each regional office chief examiner for his verification within 24 hours.

Finally, in many cases, the commission's application for an injunction and the appointment of a receiver has typically been based on violations of the net capital rules or such inadequacy of books and records that the firm is unable to make such computations as may be necessary to establish compliance with the rules concerning financial responsibility or hypothecation of customers' securities.[12]

The early warning system of the self-regulatory organizations. Over the years, the exchanges and the NASD each has developed a complex system of surveillance both on its own initiative and in compliance with Federal laws. As a result, many different early warning systems have emerged. Space does not allow a review of all of these systems individually, here. It may, however, be relatively safe to generalize that a self-regulatory organization places a member firm under a special surveillance if its net capital is deficient, the ratio of aggregate indebtedness to net capital exceeds the related rules or guidelines, or if the firm's operating losses in a given period exceed certain criteria. As an example, the New York Stock Exchange has the following early warning criteria for carrying organizations:[13]

1. Rule 325 Capital Ratio at, or in excess of 1,000 percent--as of the date of the Exchange's Early Warning Report.
2. Cumulative losses during the most recent three-month period equal to, or in excess of, 50 percent of the organization's tentative net capital.
3. Any firm likely to begin liquidation within the next six months because of losses, or scheduled withdrawals of capital, which would jeopardize their financial viability.

The final point to be noted relates to the case of NASD. The organization has recently attempted to add a new and significant feature to its operating early warning system. Due to the similarity that exists between this feature and the theme of the present study, it will be discussed in the following section in more detail. At this point, however, suffice it to say that, while all the previously-reviewed studies and the warning procedures base their analysis on the individual effects of a criterion in a selected set of criteria, the NASD's additional feature and the present work consider the collective effects of such factors. The two approaches may bring about different solutions to the same problem. But, the superiority of the second approach has been well established.[14]

As an example, consideration of a violation of the net capital rule on the basis of its individual effect may lead to a different action from that which may be induced when its effect is considered collectively, in combination with some other factor(s) such as net income, for example. As once noted by SIPC, although in a little different context,

> ...A violation of the net capital rule might not portend as serious a situation from the point of view of customer protection as originally feared. This rule basically is a test of liquidity as of a particular time. It does not necessarily follow that a temporary or possibly inadvertant failure to comply with the net capital rule makes losses to customers inevitable.[15]

The NASD Failed Firms Project[16]

When the present study was half-way through, it was found out that NASD has also undertaken a similar study, but it has not been published yet.[17] It was conducted by E.J. Dervan and E.I. Altman. It applies the Multivariate Discriminant Analysis (MDA) technique as a tool for predicting the failure of firms.

The Failed Firms Project uses a sample of two group firms: The Failed Group included 40 firms which were under SIPC liquidation; the Active Group consisted of 113 firms. None of the firms was a member of the New York Stock Exchange or American Stock Exchange. The financial data on these firms were obtained from the related Annual Financial Reports, Form X-17A-10.

The project had been launched to determine if the reported financial experience of failed firms prior to their failure could be utilized to establish an early warning system for identifying potential failures. For this purpose, it was reported that a total of 82 financial ratios and non-financial indicators such as age and type of organization were examined. The following six variables were selected as doing the best job, collectively, in discriminating between active and failed firms:

1. Net Income After Taxes/Total Assets.
2. (Total Liabilities + Subordinated Loans)/Owner's Equity.
3. Total Assets/Adjusted Net Capital.
4. (Ending Capital-Capital Additions)/Beginning Capital.

5. Scaled Age.
6. Composite (based on the ten separate variables treated in a dichotomous way).

Among conclusions of the report were that a majority of failures arise from a relatively small subset of NASD member firms; the constructed model proved to be highly accurate by classifying 90.1 percent of an original sample of failed and active firms; the model is highly reliable, and it appears to provide significant lead time in which to initiate corrective action.

The discussion about the results of the NASD study and their comparison with those of the present study is partly relegated to the next section and partly to Chapter IV, where the results of this study are presented in detail.

Differences Between the NASD Study and the Present Work

Although the two studies are similar in a number of ways, including the application of the MDA technique, they differ from each other in analytic framework, source of data, data set, and results. More specifically, the differences lie in the following areas:

The Analytic Framework

The NASD study uses the traditional approach, which may be dubbed "deterministic" to differentiate from the "probabilistic" approach being presented in the following section, in building up the prediction model. Whereas, the approach taken in the present study is a blend of "deterministic" and "probabilistic" approaches. This point will become more clear when the following section on "Different Approaches" is reviewed.

The Variable Sets

Although the complete list of the mentioned 82 variables in the NASD study has not been reported, it is most likely that the two variable sets used in building up the models differ from one another. The difference is partially due to the introduction of the "probabilistic" variables in the present study, such as standard deviation of some financial ratios and a version of Tchebycheff's inequality for estimating the probability that the value of some variables fall below a disaster level, and partly due to the sources of data which will be discussed below.

The Sources of Data

Form X-17A-10 was the source of data for the original model of the NASD study. This form contains most of the financial data which are ordinarily included in Income Statement and Balance Sheet. The present study, however, has used information contained in Form X-17A-5. This form contains only items which are ordinarily included in Balance Sheet and lacks any income-statement item. The choice of the source for the present study was inevitably due to the inavalibility of Form X-17A-10 to the public.

Data Set

In the NASD study only an original sample has been employed and it has consisted of 40 failed firms and 113 non-failed firms. In the present study, two independent samples, i.e., the original sample and the holdout sample, are used; each sample has 30 failed firms and 30 non-failed firms.

The Results

It is very difficult, if not impossible, to compare the results of the two studies. The first reason is the lack of the very important financial elements, i.e., the income-statement items, in the source of data of the present study. The second reason is the absence of similar counterparts in the two studies, especially with respect to the model which predicts the bankruptcy two years in advance. The third, but not the last, is the lack of the holdout sample in the NASD study which might have been due to the nonexistence of sufficient numbers of observations at the time of the study. Given these limitations, a comparison will be made in Chapter IV, wherever it is feasible.

Different Approaches

In predicting the financial position of a firm in a near future, there are at least three potential approaches: The first one, which has already been put to use, is financial ratio analysis, previously on the univariate basis and recently on the multivariate scale. The approach may be dubbed deterministic. The second one, which has not yet been developed is a probabilistic approach. The reasons for considering such an approach, along with a brief illustration of the concept, will be given in the following paragraphs. Then, the difficulties associated with its implementation are discussed. As a consequence, the third potential approach, which is a compromise between the first and second, emerges. This last approach is the one which has been adopted in the present study.

The Deterministic Approach

In the early years of the 1930's, Smith and Winakor[18] undertook a study of 133 financially troubled companies and concluded that Working Capital/Total Assets, Net Worth/Total Debt, and Net Work/Fixed Assets were effective indicators of financial difficulties in firms. In 1931, in another study, Fitzpatrick[19] employed 13 different ratios and concluded that all the ratios predicted failure to an extent, and that the best predictors were Net Worth/Total Debt and Net Profit/Net

Worth. Similar efforts were later made by Merwin,[20] Tamari,[21] and Beaver.[22] But, none of these researchers came to the same conclusion. What they had in common, however, was their approach to the problem which was based on the univariate analysis of the financial ratios.

Later, in 1967, Altman[23] improved upon the above approach by taking advantage of the properties of the multivariate discriminant analysis technique. This technique, which drastically reduces the analyst's space dimensionality and automatically assigns appropriate weights to appropriate variables, removed some of the ambiguities, confusions, and difficulties in the interpretation of the traditional ratio analysis. Since then this approach has been utilized in predicting the financial position of firms in various industries.[24]

Although the recent method is both theoretically and operationally superior to the traditional ratio analysis, it still shares some drawbacks with the latter: Both methods treat the process in such a way as though the property of cause and effect relation perfectly holds in a deterministic manner. The implied assumption is that a specific stimulus or a set of stimuli (i.e., changing the financial ratios) produces a specific response from the firm (i.e., failure). And, the required effort to predict the firm's state is solely to identify such stimuli. Furthermore, it is implicitly assumed that the firm is in isolation and no external influence (such as changes in the market and economic conditions) affects the future financial position of the firm. In other words, the implied assumption is that the state of the firm at some time in the future is deterministically ascertained by its current and past states.

In the face of widely prevalent uncertainty around every aspect of the business world, it is obvious that reliance on such far-out-of-reality assumptions is not plausible. In fact, the firm is a system that can be resembled to a "grey box," about which we have only a limited knowledge. The numerous subsystems, including each individual, with his own capricious and ever-changing characteristics, constituting the firm, and numerous systems and subsystems building up the socio-economic environment in which the firm exists makes any certain inference or prediction about the firm's behavior very difficult, if not impossible. The complexity of actions and interactions between these many systems and subsystems are such that we may never be able to assert with certainty that a specific input to the firm yields a specific output. Under such conditions, the best way that we may expect is a vector of possible outcomes (rather than a single certain outcome) with some probabilities attached to each of them.

The Probabilistic Approach

This approach, which has not been developed yet, is based upon the same concept which drove the statistician to replace the complexity by uncertainty. The statistician who knew that in tossing a coin the prediction of the ultimate position of the coin is conceivably possible by using the laws of mechanics (given sufficient information about its initial position, the way in which it is thrown, and the specifications of the surface on which it falls) concluded that it is far too difficult a problem for contemporary science to be solved in this way. He therefore suggested a more operational way of predicting the result of the tossing--the 0.5 probability of getting heads if the coin is a fair one.

Yet, the prediction of the firm's financial position is not as simple as that of a fair coin. The firm's position is a combinatorial phenomenon which is the result of yet-unexplored special combinations of numerous internal and external (to the firm) factors, each of which has a special probability distribution of outcome. The concept, nevertheless, is straightforward.

To illustrate, suppose that the probability of the firm's financial distress is affected by only three probabilistic factors A, B, and C, whose a priori probability distributions may be estimated through past experiments. If these three factors were mutually exclusive, the probability of financial distress would equal to the probability of getting any one of these three events. That is:

$$P(FD) = P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) \qquad (1)$$

If A, B, and C are not mutually exclusive, then equation (1) would not hold. Depending upon the anticipated influence of these factors on the probability of financial distress, there are some other choices available. If we have reasons to believe that each factor has effects *individually*, the more relevant equation may be the following:

$$P(FD) = P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \,\&\, B) - P(B \,\&\, C) - P(A \,\&\, C) \qquad (2)$$

On the other hand, if the available evidence suggests that the effect comes through the *joint* occurrence of all of the three factors, then, equation (2) does not explain the case. If A, B, and C are statistically independent, the proper equation may be the following:

$$P(FD) = P(A \,\&\, B \,\&\, C) = P(A)P(B)P(C) \qquad (3)$$

Otherwise, equation (4) may be more relevant:

$$P(FD) + P(A \,\&\, B \,\&\, C) = P(A)P(B|A)P(C|A \,\&\, B) \qquad (4)$$

The crux of the problem lies in the fact that we know neither all of the probabilistic factors affecting the probability of financial distress, nor the relationships between such factors. While some of these factors, such as long-term debt and short-term debt, may be mutually exclusive, other factors such as revenues from different product lines may not necessarily be so, and still other factors such as cash-flow and expenses may be statistically dependent. Furthermore, it is conceivable that the conditional probabilities of two factors may not be equal, i.e. $P(A|B) \neq P(B|A)$. Such a case may arise when A represents, for example, the revenue of the firm and B the general level of economic activities. Thus, the relevant probability equation of the firm cannot be a simple equation such as any one of those mentioned above. Conversely, it is likely that the function is a combination of a great number of such equations, embracing a wide range of relevant factors.

The Blend Approach

Although the probabilistic approach seems to be intriguing, it requires a great deal of research before it can be actually put to work. More specifically, the research needed comprises: (1) identifying all the probabilistic factors affecting the probability of the firm's financial distress. (2) determining the inter-relationships between these factors. (3) finding a suitable way to estimate the *a priori* probabilities of such factors, and finally (4) formulating the probabilistic model.

Faced with such a huge work, one may think about the ways of reducing the load. One way to do this, is to bridge between the deterministic and the probabilistic approach. And this is the path taken in the present study.

While the blend approach retains all the favorable properties of the deterministic approach, especially with respect to the utilization of the MDA technique, it also incorporates some facets of the probabilistic approach. Such a midway selection may ultimately be helpful in the development of the potentially more comprehensive probabilistic approach.

The Purpose of the Study

The purpose of this study is three-fold:

1. To identify potential economic and non-economic factors which might differentiate between financially distressed securities brokerage/dealer firms and those which appear to be economically viable.
2. To statistically determine a set of variables, from among the factors in point 1, which has the highest discriminatory power.
3. To suggest both public and managerial action which might be taken in light of the statistical results obtained in point 2.

The Problem Encountered

When the work on the present study had been initiated, it was hoped that no serious problem would be encountered with respect to the data needed to test the suggested hypothesis. Unfortunately, however, when the work on the theoretical part of the dissertation was finished, and the time for collecting the data came, the hope was replaced by regret.

The Public Reference Office of the SEC, although cooperative in supplying the financial Forms X-17A-5 of the selected companies, denied the accessibility of Forms X-17A-10, on the ground that they were confidential.

Since Form X-17A-5 contains only items similar to those of Balance Sheet and lacks about half of the highly valuable financial information which is contained in Form X-17A-10, especially with respect to the revenue and expense items, a full test of the suggested hypothesis is not possible with the available data. It is hoped that in evaluation of the results of the study, this limitation is kept in mind. The results might have been better if all the financial information could have been included in the analysis.

The Organization of the Dissertation

After the introductory chapter, the analytic framework is set forth in Chapter II which discusses the suggested hypothesis and determines the potential factors affecting the probability of financial distress by expanding the hypothesized function. Included in this chapter is a section on risk which explores the possible measures of risk and risk factors.

Chapter III provides a groundwork for the use of the MDA technique in the analysis. It presents an overall picture of the MDA technique in application in a summary form.

Chapter IV integrates the materials included in Chapters II and III into an empirical experiment consisting of Test I and Test II. This chapter describes the sources of data and data set, the procedure used in selecting samples, and the research methodology employed to empirically test the hypothesis. The greater part of this chapter is devoted to Test I.

Finally, Chapter V comprises a summary of the dissertation, the conclusion reached by the empirical experiment, the implication of the results found, and the suggestions for further research.

NOTES

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10. This section draws from SIPC, Third Annual Report 1973, which contains an excellent review of the manner in which SIPC's activities mesh with existing regulatory and self-regulatory organizations and procedures (see pp. 6-8 and 18-19 of the report).

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CHAPTER II

THE GENERAL ANALYTIC FRAMEWORK

Introduction

This chapter presents an analytic framework for identifying potential determinants of a firm's financial distress. The analytic framework is presented here in a general form, potentially applicable to all profit-oriented private organizations. Its application to securities brokerage/dealer firms will be discussed in Chapter IV.

The analysis begins with the hypothesis that the probability of financial distress of a firm is a function of net cash-flow per dollar of assets, risk exposure, and mismanagement of the firm. By decomposing the cash-flow equation, internal and external factors affecting revenue and expenses are examined in some detail. Risk is then defined, and after discussion of several measures of risk, risk factors associated with different facets of a firm are explained. Finally, a brief discussion is given of factors comprising the "mismanagement" element in the hypothesized function.

The analysis results in a widely expanded general function that contains a large number of factors, each of which possibly affects the probability of a firm's financial distress. What is needed, however, is a parsimonious function which contains only a few important factors with specific relations among them. Such a desired specific function may be then derived from the expanded general function by utilizing the multivariate discriminant analysis described in Chapter III.

The Hypothesis

Every industry has its own problems and peculiarities. So does each firm within each industry. Machine tool companies suffer from significant swings in scales, steel companies undergo labor strikes, and oil companies are highly dependent on geological discoveries. Food companies demonstrate sales stabilities, public utilities are regulated by governments, and business machine companies are characterized by growth.

In such diverse surroundings any specific analytic framework for identifying potential determinants of a firm's financial distress which fits a certain industry will not be of much help to another industry. What is needed, then, is a general analytic framework which may be applicable to any profit-oriented private organization, large or small, manufacturing or servicing.

The effort undertaken in this chapter hopefully will meet such a need. It is necessary, however, to treat most factors as being equally important. This is due to the fact that a factor which is relatively less important in one industry may be of greater importance in another one. Such a treatment, however, may not be harmful, because, in applying the analytic framework to a specific industry, appropriate weights will be assigned to the related factors.

To start the analysis, it is hypothesized that the probability of financial distress of a firm at time t, $P(FD_t)$, is a function of the net cash-flow per dollar of assets at time t ($CASHFA_t$), the extent of the risk assumed by and imposed upon the firm (RISK), and the mismanagement of the firm (MISMGT).

$$P(FD_t) = f\ (\tilde{CASHFA}_t,\ RISK,\ MISMGT) \qquad (1)$$

The inclusion of these elements in equation (1) is the consequence of the following line of reasoning:

With respect to the net cash-flow per dollar of assets, it is both an index of the profitability of the firm and that of the availability of funds to meet the fixed obligations in time. This index, which has been relieved from size effect of the firm, can in fact be divided into two parts, the first part being the rate of return on equity capital and the second part being a fund ratio added to or deducted from this rate due to the changes in assets (including decrease in the amount of fixed assets and increase in cash due to the depreciation allowances or assets sales) and liabilities.

The rate of return on equity capital should have a direct effect on the probability of financial distress. If the firm has a high rate of return (compared with either its cost of capital or the market risk-free rate of interest), it has a lower probability to become financially distressed than otherwise, other things held constant. The reason is that the high rate of return indicates that the firm, in addition to wholly or partially satisfying the expectations of the equity owners, depending on the height of the rate and the level of the risk, has been able to meet the requirements of the owners of debt and preferred capital, thus alleviating situations which threaten the existence of the firm. By contrast, a low rate of return, and in extreme, a negative rate of return, signifies impending inability of the firm to meet its obligations and/or probable depletion of the equity capital in shock absorbing the adverse situations. Such situations may develop very rapidly in some industries such as the securities business where the value of most assets is subject to the changes in market and business activities and the ratio of equity to debt capital is as high as 1 to 15. Therefore, the lower is the rate of return on equity capital, the higher is the probability of financial distress, other things being equal.

Although the rate of return on equity capital indicates the profitability of the firm, there have been occasions that the profitable firm (as indicated by this rate) has been unable to meet its fixed obligations in time, due to unavailability of sufficient cash in its account. On some other occasions the reverse has been observed. That is, a temporarily unprofitable firm which has had access to sufficient funds has escaped from financial distress.

Therefore, it is expected that cash-flow per dollar of assets has a higher impact on the probability of financial distress than the corresponding rate of return on equity.[1]

Regarding the risk element in equation (1), it is a complex factor which reflects the overall risk exposure of the firm. It is a combination of the risk factors assumed deliberately by the firm itself and those imposed upon it by external forces. The factors range from those associated with liquidity, borrowing and lending, to business activities, and to changes in market and the economy in general. This element will be discussed later in more detail. But, at this point, it is necessary to present the reason of its inclusion in equation (1).

By any relevant definition of risk in this context, one would expect to observe some unfavorable, as well as favorable, impact of risk factors on the performance of the firm. Thus, it seems to be relevant to relate the probability of financial distress to risk. To illustrate the point, consider two such risk factors, i.e., variance of the rate of return and financial leverage.

Since the income of a firm is subject to the changes in the level of general business activities (as well as to many other factors), its rate of return is a random variable. Therefore, the rate of return may vary from a low point to a high point over time. If the range of this variation is very wide, other things being equal, there is a higher probability of facing financial distress, as a result of several consecutive occurrences of low rates, than otherwise.

Similar reasoning applies for the dependency of the probability of financial distress upon the financial leverage. The owners' equity capital acts as a shock absorber in a car. If the equity capital constitutes a larger part of the total capital of the firm, there is a broader base to support the burden of adverse situations. That is, when several consecutive losses occur, the fixed charges of debt capital and other liabilities can be paid through digging into the equity capital. But, if the equity portion of the capital is not sufficiently large, such obligations cannot be met adequately.

With respect to the mismanagement element (MISMGT) in equation (1), the word may not suitably explain the purpose. It was, however, intended to embrace and describe all factors, such as fraud and poor books and records, which do not fall within the domain of the other two elements in the equation but have some impact on the probability of the firm's financial distress.

In equation (1) the relationships between the independent and dependent variables were considered only in a general form. It is, however, desirable to obtain at least the approximation of the true relations in a specific form. Since each of the above independent variables is under the influence of numerous factors with some possible complex interactions among them, error in computing the approximation through their direct measurements (which is only possible for CASHFA) may highly be magnified. It seems, therefore, more appropriate to break down the general relations, as far as possible, into several simpler ones. This may be done by expanding equation (1) in terms of all factors which seemingly underly the independent variables. Then, measuring each of these factors and statistically analyzing their joint effects on the probability of financial distress may lead to a better result.

Net Cash-Flow per Dollar of Assets

Starting the expansion of equation (1) by decomposing net cash-flow per dollar of assets at time t, it is known that, by definition, $CASHFA_t$ is the ratio of the available net cash-flow at time t, ACF_t, to corresponding assets, $ASSET_t$. Thus:

$$CASHFA_t = ACF_t/ASSET_t \tag{2}$$

Available cash-flow itself, however, is determined by several other factors.[2] To be more exact:

$$ACF_t = (NIAS + DEP) - GASSET + GLIAB \tag{3}$$

where NIAS = Net income available to shareholders,
DEP = Accounting depreciation and depletion allowances put aside from operating income to compensate the part of the assets used up in generating the revenue,
GASSET = Growth in assets, and
GLIAB = Growth in liabilities.

From equation (3), it is seen that available cash-flow is directly related to net income available for shareholders as well as to depreciation allowances. An increase in the assets, however, would have a negative effect, whereas a decrease in the assets has a positive effect on the available cash-flow. In contrast, an increase in the liabilities would have a positive effect, while a decrease in the liabilities has a negative effect on the available cash-flow.

In order to be able to continue the analysis, first equation (2) is restated in the following general form:

$$CASHFA_t = g(NIAS, EQC, GASSET, GLIAB, DEP) \qquad (4)$$

Subsequently observe that net income is a function of revenue (REV) and expenses (EXP).

$$\tilde{NIAS} = h(\tilde{REV}, \tilde{EXP}) \qquad (5)$$

Revenue

Revenue is the bloodstream of the firm. It is, however, subject to numerous internal and external factors. These factors may be grouped accordingly under INTREV and EXTREV variables, respectively, as in equation (6).

$$REV = i(INTREV, EXTREV) \qquad (6)$$

Internal factors. The elements of this group consist of factors which are more or less under control of the firm. When managed appropriately, these factors have positive effects on revenue, otherwise, their impacts are negative. Such factors include the following:

Number of product lines (NPL). As number of product or service lines increases, even though they may not be complementary, they reach a broader market, thus causing an increase in sales and presumably more revenue, and vice versa.

Number of places it occupies (NOPLOC). An explanation similar to that made for the number of product lines applies to the number of places that a firm occupies.

Number of years in the business (NYR). The number of years that a firm has been in the business should play a role in its profitability. Nothing can replace the experience obtained in practice. An established firm is more likely to be a profitable one. Its mere lasting existence is the evidence of its being profitable. The relationship, however, may not be linear.

Research and development (R&D). In some industries, research and development programs play a vital role in revenue generation. More importantly, in many instances, the mere existence of a firm in some industries depends on the extent to which it focuses on R&D acitivites.

Number of patents (NOPAT). The number of patents and franchises that a firm has in its possession likely affects revenue. These factors give a monopolistic power to the firm having them, and leads to potential exploitation.

Cash availability (CASHAV). Although availability of cash is an absolute necessity to run most types of businesses, and even in some instances taking advantage of possible opportunities depends on this factor, excess cash bears an opportunity cost. The income which could be earned by putting the excess cash into a productive use, is the associated opportunity cost. Leaving the excess cash idle does not contribute to the profit; rather it deducts from the potential revenue.

Securities marketability (SECMKT). Marketability of some securities is a feature which provides the possibility of replacing a part of the required cash, which brings no direct yield, with securities such as US government bonds, which bring some additional yield at practically no extra risk. If, however, the amount of this item is in excess of what is necessary, in relation to cash and other assets, the cost of unripe selling to obtain cash, or the differential yield forgone by not putting the excess amount of securities into a more productive investment, is simply a deduction from actual or potential revenue, respectively.

Credit easiness (CREDIT). A mild credit extension may improve sales, but over-easiness may bring about such large bad-debt accounts and costs that it may actually over-offset the increment revenue.

Inventory movement (INVENT). Similarly, a balanced inventory may facilitate sales, whereas too much investment in this item deducts from potential profit.

While the effects of CASH and SECMKT on REV are negative after some point, the relations of CREDIT and INVENT to REV are positive over a range.[3]

Promotional efforts (PROMOT). Another factor which is under control of the firm and has some effect on revenue is promotional efforts. The relationship, however, is a fuzzy one. The number of psychological and environmental factors which act and react simultaneously is so many that it makes clear-cut inferences too difficult, if not impossible. Yet, the large promotional expense incurred every year in all industries is an indication of its positive effect in practice.

Accounts payable (ACCPAY). Accounts payable normally contribute to revenue. This is due to the potentially better use of the funds relieved by such accounts. Usage of accounts payable, however, is not without limitations. It can reach a point beyond which its associated costs over-offsets its potential benefits.

Debt capital (DBTCAP). In the world of existing taxes, the tax treatment of interest expenses encourages greater use of debt capital. If the firm is operating profitably and the rate of interest on its debts remains constant or rises slightly as the debt is increased, the higher is the leverage, the higher becomes the profit. Even under the above conditions, however, debt may not always be profitable. Protective-covenant constraints such as a minimum working capital, a percentage limitation of long-term debt to working capital, and capital expenditure restrictions, which are imposed by debt owners, may have a significant influence on the firm's profits. Several theories have been advanced as to the optimal level of debt.[4] The controversy about them has not, however, been solved yet.

Debt mix. The optimal mix of short-term (SHRTDT) in relation to long-term (LONGDT) financing is also of considerable importance. This is due to both interest rate differential and other factors such as flexibility, timing and etc.[5]

There are of course some other internal factors, but their effects on REV, individually and perhaps collectively, is likely to be minor, relative to the previously-mentioned factors. Therefore, the above discussion may be summarized in the following equation:

$$\begin{aligned} \text{INTREV} = j(&\text{NPL, NOPLOC, NYR, R\&D, NOPAT, CASHAV,} \\ &\text{SECMKT, CREDIT, INVENT, PROMOT, ACCPAY,} \\ &\text{DBTCAP, SHRTDT, LONGDT}) \qquad (7) \end{aligned}$$

The relative importance of each of the above factors in the profitability of a firm differs from one industry to another and within subdivisions of each industry. Therefore, the explanatory variables for describing the impacts of such factors on the probability of financial distress of the firm should fit to specific industry and its subdivision to which the firm belongs.

External factors. The elements of this group consist of factors over which the firm does not have any control, yet they have some effects on the revenue of the firm. Among these factors, general business activity has a special place.

General business activity (GBA). General business and economic activity is subject to recurrent but non-periodic fluctuations, called business cycles, over a long period of time. These fluctuations occur in aggregate variables such as prices, GNP, investments, employments, and income. The variables move at about the same time and in the same direction, but their rates of change are different. Although the effect of these fluctuations on a firm differs from one industry to another, no firm remains immune from their impact. In general, over any period of time, the revenue of a firm is a function of the level of general business activity.

Interest rate (INTRR). The market prime rate of interest has also some effects on the revenue of the firm. This, of course, relates to the lending position of the firm. For a firm which has a borrowing position the effect of interest rates on its performance comes through expenses. If the firm is both lender and borrower at the same time, the effect comes through both revenue and expense sides. The net effect, then, may be either positive or negative, depending upon the net lending or borrowing position of the firm.

Population (POPUL). Population is another important external factor. Not only the number of inhabitants in the related area matters; the population mix has also bearing on the revenue. A firm which produces youth products is more profitable in an area where the young constitute a higher percentage of the population than in other areas, other things being equal.

Per capita income (PCINC). Per capita income of the inhabitants in the area where the firm runs its business is another determinant of the firm's revenue. Per capita income, which is an index of the purchasing power of the people residing in an area, differs from one region to another. The variation is due to difference in the mix of industries and in the prevalence of large cities, where wages are higher, than in small towns and rural areas.

Competition (COMP). Competition, if it exists, diffuses the potential market power of the firm. Then, the price which it puts on its products and, as a consequence, its revenue, is governed by the market. In fact, the effect of competition, which is the foundation of capitalism, is to give rewards, in the form of profits, to the firms which are able to survive, and penalties, in the form of losses or bankruptcy, to those which are not.

Competition, in a broad sense, embraces all factors of production--land (which includes all natural resources), capital, labor, and entrepreneurship. In contrast to the firm's desire to have a monopoly position in selling products and monopsony power in buying supplies and services of labor, land, and capital, the actual situation which it faces may be exactly the opposite one. Existence of labor unions, monopoly in some raw materials, branch banking systems in some states, and exclusively governmental purchases are commonplace facts which are faced by many firms. All these factors have bearings on the firm's revenue. Indeed, wherever appropriate, COMP factor should be broken down into the above four dimensions (corresponding to the factors of production) in evaluating the firm's position with respect to competition.

Regulations (REGUL). Rules and regulations are imposed by either governmental regulatory agencies or self-regulatory organizations. They are intended to regulate various aspects of the activities of firms in different industries. These rules and regulations significantly affect the firm's profit. Rules and regulations governing the rates of public utility companies and railroads can be mentioned as examples. The rules and regulations sometimes go to extremes. In banking, for example, the regulations require a relatively low marginal rate of return on capital. Each industry has a set of regulations relevant to itself. Therefore, in evaluating the position of the firm in a certain industry with respect to the prevalent rules and regulations in that industry, factor REGUL should be broken down into an appropriate number of dimensions.

The above discussion can be mathematically expressed:

$$EXTREV = k(GBA, INTRR, POPUL, PCINC, COMP, REGUL) \qquad (8)$$

Expenses

The expense element (EXP) in equation (5) may also be broken down into the part originated by internal factors (INTEXP), and that caused by external factors (EXTEXP). Thus:

$$EXP = l(INTEXP, EXTEXP) \qquad (9)$$

Internal factors. Most of the internal factors affecting the overall expenses of a firm can be categorized into selling expenses (SELEXP), general expenses (GENEX), and administrative expenses (ADMEXP). Although all firms have these three types of expenses, the relative importance of each type differs from one industry to another. Even in the same industry the percentage of each type differs among firms. This very difference may differentiate between firms operating optimally or near optimally and those which are run sub-optimally. In fact, there can always be found one or more expense items which are conspicuous in a specific industry. As an example, the communication expenses of firms in the securities industry is prominent. Evaluation of a firm's position with respect to such an item may reveal some characteristics of the firm's performance. To obtain such items, it is useful to break down the above three types of expenses into more detailed components peculiar to each industry. In any event, depreciation expenses, which may belong to one or more of the above categories, should be singled out for calculating the net cash-flow, as stated in equation (3). To summarize the above discussion:

$$INTEXP = m(SELEXP, GENEXP, ADMEXP) \qquad (10)$$

External factors. Although a little fuzzy, all external factors which affect a firm's profit other than through the revenue side fall in this category. One of the most important elements in such factors is taxes.

Taxes (TAX). Aside from the economic purposes of taxes, i.e., stabilization of economy, redistribution of wealth, and reallocation of resources, businessmen look at them as a compulsory payment to government. Indeed, they reduce the firm's profit by a large percentage, called tax rate. The legal tax rate varies over a period of time. Whenever the rate is lower, the income remaining for shareholders is larger, other things held constant, and vice versa.

Interest rate (INTRR). The role of the interest rate in the performance of the firm was discussed in the revenue section. To avoid exclusion of such an important factor from among the determinants of expenses, however, it is reintroduced in this section as well. In fact, since debt usually constitutes a main part of the capital structure of most firms, the impact of the interest rate coming through the expense side seems to be more important for most firms. In any event, the higher is the interest rate, the larger is the interest charge as part of the firm's total expenses, and vice versa.

Environmental factors (ENVIRN). Other factors such as anti-pollution expenses, benevolence expenses, and the like may be grouped under environmental factors. Firms manufacturing defense products may incur opportunity costs. These costs, which are equal to the differential profit forgone by not employing the resources in a more productive way should rightly be considered as expenses. In summary:

$$EXTEXP = n(TAX, INTRR, ENVIRN) \tag{11}$$

Risk

It is now necessary to refer back to equation (1) and make an effort to identify the components of the overall risk of the firm which may play important roles in determining the probability of financial distress. Before going into details, however, a digression on definition and measures of risk seems appropriate.

Definition

Risk has been defined differently in various sources, both in view of context and in regard to content. In Webster's Dictionary, it has been defined as the possibility of loss, injury, disadvantage, or destruction.[6] Sloan and Zurcher defined risk as the possibility of loss from some particular hazard, as fire risk, war risk, credit risk.[7]

In the McGraw-Hill Dictionary, risk has been defined as the exposure of an investor to the possibility of gain or loss of money.[8] Machol and Lerner defined risk as the cumulative probability of the return falling below some level of ruin.[9] And, in the state-of-nature approach to evaluating risk, it is implicitly defined as the possibility that the desired, rather than the expected, return will not be achieved.[10]

Although all of these definitions suit their own places, there is need for some definition which fits the context under study. For that purpose, risk is here defined as the possibility that actual outcome of a random variable[11] will deviate from that which is expected. The random variable can, of course, represent such diverse factors as cash-flow, rate of return, expenses, bad debts, or even the business activity level.

Common Measures of Risk

Having risk thus defined, then, a measure of risk is a means by which the extent of risk is ascertained. There is, however, no single measure of risk which can well explain the actual risk associated with any factor under consideration. Facing this difficulty, the authors in the field of economic and business, and especially in the porfolio literature, have adopted various means as measures of risk, each with its own virtues and limitations. So far, the most popular measure of risk has been variance (or standard deviation) of a random variable around its mean.

Variance or standard deviation of a single variable. Tobin, in adopting the standard deviation as a measure of risk, although in a portfolio context, made the following statement in his celebrated paper on "Liquidity Preference as Behavior Towards Risk:"[12]

> The risk attached to a portfolio is to be measured by the standard deviation of R, σ_R. The standard deviation is a measure of the dispersion of possible returns around the mean value μ_R. A high standard deviation means, speaking roughly, high probability of large deviations from μ_R, both positive and negative. A low standard deviation means low probability of large deviations from μ_R; in the extreme case, a zero standard deviation would indicate certainty of receiving the return μ_R. Thus a high-σ_R portfolio offers the investor the chance of large capital gains at the price of equivalent chances of large capital losses. A low-σ_R portfolio protects the investor from capital loss, and likewise

> gives him little prospect of unusual gains. Although it is intuitively clear that the risk of a portfolio is to be identified with the dispersion of possible returns, the standard deviation is neither the sole measure of dispersion nor the obviously most relevant measure....

The variance of a random variable is the expected value of the squared deviations from the mean:

$$V(R) = \sum_{i=1}^{n} (R_i - \overline{R})^2 P_i \tag{12}$$

where V(R) is the variance of the random variable R, n is the number of possible outcomes, R_i is the i-th possible outcome, $\overline{R}$ is the expected value of all possible outcomes, and P_i is the probability or the relative frequency, whichever is appropriate, of occurrence of the i-th event.

The standard deviation is simply the square root of the variance:

$$\sigma_R = \sqrt{V(R)} \tag{13}$$

The variance obtained by equation (12) is in an absolute term. For comparing the variability of different distributions, however, there is need for a measure of relative dispersion. This need is met by forming the coefficient of variation as expressed below:

$$CV(R) = V(R) \ / \ \overline{R} \tag{14}$$

Semi-variance. Markowitz, who pioneered the modern theory on portfolio selection, considered the standard deviation, the semi-variance, the expected value of loss, the expected absolute deviation, the probability of loss, and the maximum loss as candidate measures of risk.[13] Among these measures, however, he concentrated more on the first two measures.[14] One of the reasons for considering semi-variance as a candidate measure of risk is its ability to measure the skewness of the probability distribution of the random variable, say, return.

Unlike the variance, which ignores the shape of the probability distribution, semi-variance takes into account the downside or upside tendency of the probability distribution. Since most economic units associate risk with the possibility of loss, downside fluctuations in return, and not dispersion per se, have considerable importance. A distribution skewed to the left, as indicated by semi-variance, would involve more chance for low or negative returns than a distribution skewed to the right, all other things being the same.

Semi-variance can be expressed in the same way as variance was stated in equation (12) with the exception that n and i now represent, respectively, only total number of those outcomes and the i-th outcome whose values are smaller (or greater, according to one's purpose) than $\bar{R}$, the overall mean value:

$$SV(R) = \sum_{i=1}^{n} (R_i - \bar{R})^2 \, P_i \qquad (15)$$

Markowitz suggested the ratio of V(R)/2SV(R) as a measure of relative skewness.[15] For symmetric distributions this ratio equals one, but it is greater or less than one if distribution skewed to the right or left, respectively.

Third moment. The measure of skewness is not limited, of course, to semi-variance. Skewness, in absolute or relative sense, may be measured also by the third moment of the probability distribution of the random variable. The third moment about the mean may be expressed by:

$$m_3 = \sum_{i=1}^{n} (R_i - \bar{R})^3 P_i \qquad (16)$$

which is similar to equation (12), with the exception that the deviations from mean, the term $(R_i - \bar{R})$, is now cubed rather than squared. Cubing the term does not change the sign of the deviations, but it magnifies the extreme deviations, thus making the risk measurement possible. Against this advantage, the third moment has the same disadvantage that other moments have, i.e., it is greatly affected by a few extreme deviations. In fact, since the absolute amount of skewness depends on the dispersion of data, a given value of m_3 could be the result of slight asymmetry together with large dispersion, or great asymmetry with small dispersion. Its usefulness cannot be doubted, however, especially when it is combined with other moments.[16]

Arditti, in an empirical study, "Risk and the Required Return on Equity,"[17] employed second and third moments along with some other risk factors,[18] and concluded, among other things, that the market required return had direct relation with third moment, as was expected from a measure of risk.

Third moment may become more useful in comparing among several distributions if it is expressed in a relative term. This is done by dividing the moment by the cube of the standard deviation of the random variable and is called standardized third moment:

$$a_3 = m_3/\sigma_R^3 \tag{17}$$

Rate of growth. It was discussed in previous paragraphs that third moment about the mean is greatly affected by a few extreme deviations. In fact, the sign of the moment may be largely attributable to a single large negative (or positive, as appropriate) deviation. Furthermore, what a measure of skewness shows is the existence of asymmetry in the frequency distribution. It can not identify how this asymmetry occurred, i.e., whether it was due to regular or irregular changes in the values of the random variable. It is obvious that a steady trend in one direction removes most of the uncertainty (or risk) associated with predicting a certain value for the variable, whereas irregular fluctuations make such prediction very difficult.

In order to detect the regular changes in one direction, the concept of rate of growth, whether positive or negative, can be utilized. There are two ways to get the rate of growth: the first one is through applying a geometric mean method. The second is through utilizing the regression analysis concept. The latter method, which is discussed below, seems to have more advantage than the former. This is due to its ability to provide some additional information, such as R^2, the coefficient of determination of the regression line.

The model which is discussed here has originated from that which is used for computing the continuously compounded rate of interest.[19] It is expressed as:

$$V_t = Ae^{rt} \tag{18}$$

where V_t is the value of the variable after t years, A is the initial value, e = 2.71828, r is the instantaneous rate of growth, and t is total years under consideration.

To calculate r, first take the natural log from both sides of equation (18):

$$\ln V_t = \ln A + rt \tag{19}$$

Then, by utilizing equation (19), regress $\ln V_t$ against t, and obtain r as well as ln A and R^2, the coefficient of determination, of the regression line. R^2, which is bounded between zero and one, is a useful measure of the goodness of fit of the regression line. The closer is R^2 to one, the better is the fit.[20]

In analyzing risk, it seems more useful to use R^2 along with r, the rate of growth, and both R^2 and r in conjunction with other measures of risk such as variance and semi-variance, because each of these measures captures only a portion of total risk associated with the variation of the random variable.

Confidence level. Baumol, who was also concerned about the downside fluctuations of a random variable, such as return, proposed still another measure of risk.[21] His measure of risk resembles a lower bound or confidence limit on the expected outcome and is expressed by:

$$L = \bar{R} - k\sigma_R \tag{20}$$

where L is the suggested measure of risk, $\bar{R}$ is the mean of the random variable, k is a positive parameter that represents the number of standard deviations below the mean value, and σ_R is the standard deviation. Unlike the previous measures of risk, however, this one is not independent of the evaluator's preferences. That is, before any value can be found for L, a value should be assigned by the evaluator to k. The dependence of this measure of risk on the evaluator's utility function detracts from its usefulness for the purpose, here.

Covariance. One of the useful measures of risk is covariance between two random variables, say, the firm's net income and the changes in business activity. It is a measure of the extent to which the value of the two variables tend to move up and down together. It can be expressed by the following equation:

$$COV(R,Q) = \sum_{i=1}^{n} (R_i - \bar{R})(Q_i - \bar{Q}) \; P_i \qquad (21)$$

where R and Q are two random variables with expected values of $\bar{R}$ and $\bar{Q}$, respectively, R_i and Q_i are the i-th possible outcomes of R and Q, n is the total possible outcomes of each variable, and P_i is the joint probability that R_i and Q_i will occur simultaneously. As is seen from equation (21), when both R_i and Q_i are above or below their related means for most of the times, the covariance becomes positive, whereas, when one of them is above its mean more frequently while the other is below its mean for most of the times, the covariance becomes negative.

Correlation coefficient. Although the covariance formula is a good mean to explain the relative movements of the random variables, its absolute value is difficult to interpret. A better measure which is directly related to covariance and can be easily interpreted is correlation coefficient. It may be expressed by:

$$COR(R,Q) = \frac{COV(R,Q)}{\sigma_R \cdot \sigma_Q} \qquad (22)$$

If the two random variables move up and down in perfect unison, the correlation coefficient is one (they are perfectly correlated). A correlation coefficient of zero, on the other hand, indicates that their movements are completely independent from each other (they are uncorrelated). If the two variables move in reverse direction in perfect unison, the correlation coefficient is minus one (-1). Any other imperfect correlation lies somewhere between +1 and -1.

Variance of a group of variables. In the portfolio context, where the return on portfolio is dependent upon the returns on the constituent securities, it has long been known that the variance of the portfolio return is a function of the variances of the comprising securities as well as the correlation coefficients between every pair of these securities. More exactly:

$$V(P) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \, COR(i,j) \, \sigma_i \sigma_j \qquad (23)$$

where n is total number of securities in the portfolio, X_i and X_j are the percentages of funds invested in securities i and j, COR(i,j) is the correlation coefficient between these securities, and σ_i and σ_j are standard deviations of securities i and j.

The concept may be extended to contexts other than portfolio, where determination of a variation of a single random variable, such as operating revenue of a firm, in terms of variations of several other random variables, for example, revenue from each line of product, is also desirable. All elements of equation (23) can then be easily redefined to fit such cases, except for X_i and X_j which should be interpreted in a slightly different way. In retrospect, when past series data are used, one may interpret X_i and X_j as the shares of random variables i and j, respectively, in the overall picture. For example, if talking about the variation in the company's revenue, one may interpret X_i and X_j as average proportions of the total revenue which are contributed by product lines i and j, respectively.

Computation of the collective variance by equation (23) is easy when the number of constituent random variables is small. For a large number of such variables the computation in this way becomes cumbersome. The diagonal model presented by Sharpe[22] has solved this problem. In the portfolio context, the following steps should be taken to calculate the variance of portfolio:

First, a suitable market index such as GNP is selected and then by using the following linear equation for each security, individually, various returns of each security are regressed against the corresponding value of the index:

$$R_{ij} = A_i + B_i I_j + e_i \tag{24}$$

where R_{ij} is the j-th observation of the random variable i, I_j is the j-th observation of the index, A_i, B_i and e_i are y-intercept, slope, and random error related to the regression line of the variable i, respectively.

Then, by using the results obtained from this step in the following equation, variance of the portfolio is calculated:

$$V(P) = \sum_{i=1}^{n} X_i^2 \sigma_i^2 + X_{n+1}^2 \sigma_{n+1}^2 \qquad (25)$$

where n is the total number of securities in the portfolio, X_i is the percentage of funds invested in security i, σ_i is the standard deviation of e_i in (24), σ_{n+1} is the standard deviation of the index, and X_{n+1} is a "pseudo" security defined as

$$X_{n+1} = \sum_{i=1}^{n} X_i B_i \qquad (26)$$

The expected value of the portfolio, which is needed for computing the coefficient of variation, is calculated by the following equation:

$$E(P) = \sum_{i=1}^{n} X_i A_i + X_{n+1} A_{n+1} \qquad (27)$$

where A_{n+1} is the expected value of the index. The coefficient of variation is:

$$CV(P) = V(P)/E(P) \tag{28}$$

Again, these formulations may be applicable to contexts other than portfolio, as will be shown later.

The probability of disaster level. In the economic world where the outcomes of most activities are subject to uncertainty there is always a range of outcomes for each activity over which the economic unit is drastically affected. The upper bound of the range where the vulnerability to the chance event just begins may be referred to as disaster level.

Examples of disaster level can be found abundantly in the firm's various facets. If the firm's net cash-flow drops below a certain point, for example, the firm may face a hard time. When the sales shrink to a specific level, it may experience large losses. Or, if the total asset value declines to a level which is less than the total liabilities, the firm becomes insolvent.

If a means can be found to show the chance of a random variable being at the disaster level or below, then it can be used as an appropriate measure of risk for dealing with such cases. Fortunately, following Roy,[23] one can apply Tchebycheff's inequality in obtaining such probability. This, of course, assumes possession of information about the mean and variance as well as the disaster level of the random variable. The Tchebycheff inequality may be expressed as the following in this situation:

$$P(|R - E| \geq E - d) \leq \sigma^2/(E - d)^2 \qquad (29)$$

or,

$$P(R \leq d) \leq \sigma^2/(E - d)^2 \qquad (30)$$

where P is the probability operand and R is a random variable whose expected value, variance, and disaster level are, respectively, E, σ^2 and d.

One of the advantages of the Tchebycheff inequality is that it does not require the probability distribution of the random variable to be normal. There is, however, a difficulty in applying equation (30) in the context used here. This difficulty is due to the unavailability of disaster level (d) in any of the related cases. Fortunately, however, a designated d need not be very exact, because firms usually are able to manage their affairs over a range of each variable. And this range may not be relatively too narrow.

A sufficiently close approximation of d may be obtained by averaging the variable over all available firms of similar size and nature which have actually experienced the financial distress. The implied assumption here is, of course, the dependence of the financial distress upon the disaster level of the variable.

If data unavailability precludes grouping the firms of similar size and nature for this purpose, one may resort to some appropriate ratio of the variable. The ratio could first be averaged over all available firms which have experienced financial distress and the average could then be transformed to an absolute value for each firm by a simple calculation. As an example, suppose that the average disaster ratio of sales to assets turns out to be 0.5, then the disaster level of sales for a firm having a total assets of one million dollars can be obtained by multiplying one million by 0.5 which equals $500,000.

Risk Factors

Risk associated with cash-flow (RSKCF). The importance of sufficient cash inflow in meeting the firm's obligations cannot be over-stated. Since cash-flow is a random variable, at any point of time, it may take some value between two possible extreme points relevant to the firm. But, at some level of cash inflow which is of course different for each firm, the firm may not be able to run the business smoothly. The possibility that the cash inflow falls below this disaster level, then, constitutes a risk factor which may play an important role in determining the possible financial distress of the firm.

If it is possible to estimate such a disaster level for each firm, it may then be possible to estimate the probability of its occurrence by resorting to the Tchebycheff's inequality, as reproduced here.

$$P(ACF \leq d) \leq \sigma^2_{ACF}/(\overline{ACF} - d)^2 \tag{31}$$

where P is the probability operand, ACF is the available cash-flow variable whose mean value and variance are $\overline{ACF}$ and σ^2_{ACF}, respectively, and d is the disaster level for available cash-flow.

To find the disaster level of each firm it is highly desirable to have access to data of as many failed firms as possible which have the same characteristics, particularly with respect to size and business nature, as those of the firm under consideration. But, since finding such firms is not always possible, a reasonable approximation may be obtained by first averaging the ratio of ACF to total assets of all failed firms in the same industry, and then transforming this average ratio to an absolute value through its multiplication by total assets of the firm.

To find the probability that the cash-flow of the firm may fall below this calculated disaster level, it is necessary to calculate the mean value and variance of cash-flows of the firm over a recent period of time to be used, along with the calculated disaster level, in the Tchebycheff's inequality. The probability obtained in this way is a conservative one, yet it may be a useful approximation to risk assumed by the firm with respect to its cash-flow procedure.

Donaldson has proposed that the main concern of a company is to see whether the cash balances fall below zero or not, and to determine such possibility, he suggests examining the cash-flows under recession condition.[24]

If that is the case, ACF in equation (31) may be redefined to be net cash-flow. Then, the disaster level, d, changes to zero, and the probability that the net cash-flow falls below zero becomes equal to or less than the ratio of the coefficient of variation to the mean value of the net cash-flow:

$$P(ACF \leq 0) \leq CV(ACF)/\overline{ACF} \qquad (32)$$

The preference of (31) to (32), or the other way around, as a determinant of the firm's risk exposure depends, of course, on the results of empirical checks.

Risk associated with revenue (RSKREV). Risk associated with revenue is the possibility that actual revenue will deviate from that which is expected.

Following the discussion above of the measure of risk section with respect to the properties of the variance of a random variable, one measures the risk associated with total revenue by finding the variance of the values taken by revenue variable over the period under consideration.

Equation (33) may be used for such purpose:

$$V(REV) = \frac{1}{n} \sum_{i=1}^{n} (REV_i - \overline{REV})^2 \qquad (33)$$

where V(REV) is variance of revenue, n is total number of observations on revenues, REV_i is the i-th observation, and $\overline{REV}$ is the mean value of total observations on revenue. This equation provides an absolute value for the variability of revenue. Its relative value, however, which is obtained by coefficient of variation, may be more useful in discriminant analysis:

$$CV(REV) = V(REV)/\overline{REV} \qquad (34)$$

As was discussed in previous section, however, the variance alone does not capture all the risk associated with revenue. If the frequency distribution of revenue is skewed to the left, there is higher possibility of loss, and as a consequence, higher possibility of financial distress. It may, therefore, be helpful to supplement the coefficient of variation with a measure of relative skewness. For this purpose it is possible to use either semi-variance or third moment of revenue. But to choose between them by detecting the discriminatory power of each of them, it seems appropriate to use both in this analysis. The semi-variance of revenue may be expressed by:

$$SV(REV) = \frac{1}{n} \sum_{i=1}^{n} (REV_i - \overline{REV})^2 \qquad (35)$$

where n represents the number of values below the mean, $\overline{REV}$, and REV_i is the i-th observation of such values.

The relative skewness, using semi-variance, is thus obtained by the following ratio:

$$CSV(REV) = V(REV)/2SV(REV) \tag{36}$$

The third moment about the mean of the frequency distribution of revenue may be expressed by:

$$m_3 = \frac{1}{n} \sum_{i=1}^{n} (REV_i - \overline{REV})^3 \tag{37}$$

where right-hand symbols are the same as those in equation (33).

The standardized third moment which indicates the relative skewness of revenue frequency distribution is then obtained by:

$$a_3 = m_3 \ / \ V(REV)^{\frac{3}{2}} \tag{38}$$

For the reasons explained in the measures of risk section, it may be useful to consider the rate of growth of revenue in conjunction with the above-mentioned measures of revenue dispersion and skewness. It is necessary to redefine the variables in equation (19), first:

$$\ln V_t = \ln A + rt \tag{39}$$

where A is the initial total revenue, V_t is total revenue after t years, r is the rate of growth, and t is the t-th year of T, the total years under consideration.

Then, a regression line is fitted to the points obtained by plotting the natural log of total revenue of each year (V_t) against the time (t). The rate of growth and R^2 obtained from this line is then used in the analysis.

Risk associated with product lines (RSKPL). The risk associated with revenue changes with the number of product lines that the firm offers. The reason is based on the same principle applied to diversification in investment. Indeed, having several product lines may be necessary but it is certainly not sufficient to reduce risk. If all the product lines are perfectly positively correlated, with respect to revenue, with each other, then there accrues no benefit from such diversification. Their collective performance resembles that of any constituent individual line, except on a larger scale.

Negative correlations between revenues generated by each product line, on the other hand, reduce the risk associated with revenue. The higher is the degree of diversification of product lines with such properties, the lower is the related risk. The recent great wave of conflomeration[25] seems to be partially, at least, in pursuit of such concept.[26]

Existence of some negative correlations, however, does not guarantee that the collective risk is changed significantly. If the analysis of the historical data indicates that the product line with negative correlation contributes only a small percentage to the total revenue, its risk-reducing effect would be minor, and vice versa.

All the above points are accounted for if, following the portfolio concept, the variance of total revenue is expressed by the following function:

$$V(REV) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j COR(i,j)\sigma_i \sigma_j \qquad (40)$$

where n is the number of product lines, X_i, X_j are the historical average proportions of total revenue which come from lines i and j, respectively, COR(i,j) is the correlation between revenues from lines i and j, and σ_i, σ_j are standard deviations of revenues from lines i and j, respectively.

Under a certain condition, it may be possible to calculate the risk associated with product lines. This condition is that the firm should either have only one store, or else, if it is a multi-store firm, all the stores should be, revenue-wise, perfectly positively correlated to each other. If this condition is not met, however, the calculated risk represents the combined risks of both product lines and stores.

To find the risk associated with product lines, assume first that all the lines are perfectly positively correlated with each other; then all correlation terms in equation (40) would, by definition, be equal to one. This constitutes the upper limit of the variance of total revenue. On the other hand, if all the product lines were perfectly negatively correlated to each other, all correlation terms in equation (40) would, by definition, equal minus one. This constitutes the lower limit of the variance of total revenue. For any imperfect correlation, either positive or negative, between product lines, each correlation term in equation (40) assumes a value between -1 and +1. The weighted average of these correlations then determines the position of the variance of total revenue between its upper and lower limits.

Bearing in mind the above discussion, one may express the upper limit of the variance of total revenue by the following equation:

$$V(REV) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j \qquad (41)$$

The case for any position between upper limit and lower limit could, of course, be expressed by equation (40) itself. The deduction from variance of total revenue which resulted from imperfectly correlated product lines, or equivalently, the risk (although negative) associated with product lines, is then equal to the negative of the difference between equations (40) and (41).

$$RSKPL = - \left[\sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j - \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \, COR(i,j) \, \sigma_i \sigma_j \right]$$

The first term in the bracket can be rewritten as:

$$\sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j, \text{ for all } i \neq j$$

The second term in the bracket can be rewritten as:

$$\sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \, COR(i,j) \, \sigma_i \sigma_j, \text{ for all } i \neq j$$

Replacing the terms in brackets with their equivalents, omitting $\sum_{i=1}^{n} X_i^2 \cdot \sigma_i^2$, and factoring common factors, there is obtained

$$RSKPL = - \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j \, [1 - COR(i,j)] \text{ for all } i \neq j \tag{42}$$

The negative sign in front of the Σ notation indicates that multiple product lines have a risk-reducing effect, but to compare the effects of various product lines one should compare their negative effects on reducing risk associated with total revenue. The larger is the absolute value of RSKPL, the larger is the deduction from the variance of total revenue as a result of such diversification, and vice versa. Therefore, the larger is the absolute value of RSKPL, the more desirable is the product line, and in the discussed risk context, the smaller is the risk associated with it. Conversely, the smaller is the absolute value of RSKPL, the larger is the risk associated with product line. The lower bound of the absolute value of RSKPL is zero, and it occurs just when all the product lines are perfectly positively correlated, i.e., correlations between all product lines are equal to one and thus all [1-COR(i,j)] terms in equation (42) become zero. This, of course, means that no benefit is accrued from such diversification.

The risk expressed in equation (42) is in an absolute term. In order to make it in a relative term, and bring it into a line with the coefficient of variation, divide both sides of (42) by the mean value of total revenue. Thus, denoting the relative risk associated with product line by RRSKPL, there is obtained:

$$RRSKPL = -\frac{1}{\overline{REV}} \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j [1 - COR(i,j)], \text{ for all } i \neq j \qquad (43)$$

If the product lines are many and calculation of the covariance terms is cumbersome, it may be helpful to apply the indexing method in computing the risk associated with product lines. In that case, GNP or another suitable market index is selected and variables in equations (24) through (27), which are reproduced here, are redefined to fit the revenue context. Thus:

$$REV_{ij} = A_i + B_i I_j + e_i \qquad (44)$$

where REV_{ij} is the j-th observation of the revenue generated by product line i, I_j is the j-th observation of the index, and A_i, B_i, and e_i are y-intercept, slope, and random error related to the regression line of the product line i, respectively.

$$V(REV) = \sum_{i=1}^{n} X_i^2 \sigma_i^2 + X_{n+1}^2 \sigma_{n+1}^2 \qquad (45)$$

where V(REV) is variance of total revenue, σ_i^2 is the variance of e_i, σ_{n+1}^2 is the variance of the index, and X_{n+1} is "pseudo" product line defined as

$$X_{n+1} = \sum_{i=1}^{n} X_i B_i \qquad (46)$$

And the mean value of total revenue is obtained by:

$$\overline{REV} = \sum_{i=1}^{n} X_i A_i + X_{n+1} A_{n+1} \qquad (47)$$

where A_{n+1} is the expected value of the index.

Now, if it is assumed that all the product lines are prefectly positively correlated to each other, then $B_1 = B_2 = \ldots = B_n = B^*$, where B^* is the slope of the regression line fitted to past series data on total revenue and the index values. In that case:

$$X_{n+1} = \sum_{i=1}^{n} X_i B_i = B^* \sum_{i=1}^{n} X_i = B^* \qquad (48)$$

Replacing (48) in (45), gives the risk associated with total revenue in the absence of any advantage from diversification.

$$V(REV) = \sum_{i=1}^{n} X_i^2 \sigma_i^2 + B^{*2} \sigma_{n+1}^2 \qquad (49)$$

Reduction in this risk as a result of combining product lines which are not perfectly positively correlated to each other is obtained by calculating the difference between equations (49) and (45). Again, the negative of this difference is the risk associated with product lines.

$$RSKPL = - [\sum_{i=1}^{n} X_i^2\sigma_i^2 + B^{*2} \sigma_{n+1}^2 - (\sum_{i=1}^{n} X_i^2\sigma_i^2 + X_{n+1}^2\sigma_{n+1}^2)]$$

$$= - \sigma_{n+1}^2 (B^{*2} - X_{n+1}^2)$$

or,

$$RSKPL = -\sigma_{n+1}^2 \; [B^{*2} - (\sum_{i=1}^{n} X_i B_i)^2] \qquad (50)$$

To get the relative risk associated with product lines, divide both sides of equation (50) by the mean value of total revenue.

$$RRSKPL = - \frac{\sigma_{n+1}^2}{\overline{REV}} \; [B^{*2} - (\sum_{i=1}^{n} X_i B_i)^2] \qquad (51)$$

Risk associated with places occupied (RSKPO).

Another factor which may have risk-reducing effect on the firm's position, is the number of places that it occupies. The effect comes through the same principle as applied to product lines. Over any period of time, many factors, such as changes in the competitive forces, changes in the buying population, and changes in the purchasing power of the inhabitants of the location where the firm has its business act and react on the firm's overall performance.

Supposedly, diversification into several places deducts from the dependency of the firm's performance on a set of factors peculiar to a certain location. This may mean reduction in the business risk of the firm, but this reduction may not materialize if some conditions, similar to those of product lines, do not exist. To name only a few, these conditions include existence of negative correlations between each pair of stores, as well as the relative importance of the contribution that the stores with negative correlations make to total revenue.

It seems also plausible to expect that the reduction in risk through multiplication of stores is, ordinarily, in addition to that which is resulted from diversification in product lines. Otherwise, the multiplication of stores would have a negative effect.

The provision for calculating risk associated with number of stores is similar to that for product lines. That is, the firm should either have only one product line, or else, all product lines should be perfectly positively correlated to each other.

To calculate RSKPO, the procedure is the same as that applied to the product lines case. First, suppose that the stores in different places are perfectly positively correlated with each other. Thus:

$$V(REV) = \sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j Q_i Q_j \qquad (52)$$

where V(REV) is variance of total revenue, n is the number of stores, and Y_i and Y_j are the historical average proportions of total revenue which come from stores i and j, respectively, and Q_i Q_j are standard deviations of revenues from stores i and j, respectively.

Then, consider the case in which the stores are not, revenue-wise, perfectly positively correlated with each other. In that case:

$$V(REV) = \sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j COR(i,j) Q_i Q_j \qquad (53)$$

where COR(i,j) is the correlation between revenues from stores i and j.

The deduction from variance of total revenue as a result of the imperfectly correlated stores is the difference between equations (53) and (52). Thus, the risk associated with stores (although negative) is the negative of such difference:

$$RSKPO = - \left[\sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j Q_i Q_j - \sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j COR(i,j) Q_i Q_j \right] \qquad (54)$$

This expression, after some manipulation similar to that in the product lines case, can be rewritten as the folllowing:

$$\mathrm{RSKPO} = - \sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j Q_i Q_j \, [1 - \mathrm{COR}(i,j)], \text{ for all } i \neq j \qquad (55)$$

Which can be put in a relative form by dividing by the mean value of total revenue:

$$\mathrm{RRSKPO} = - \frac{1}{\overline{\mathrm{REV}}} \sum_{i=1}^{n} \sum_{j=1}^{n} Y_i Y_j Q_i Q_j [1-\mathrm{COR}(i,j)], \text{ for all } i \neq j \qquad (56)$$

At this point, it should be noted that when the firm has more than one product line, or, otherwise, when the product lines are not perfectly positively correlated to each other, the calculated RSKPO above represents the total risk associated with diversification in both dimensions, product lines and stores. For such companies, the value of this combined risk should conceivably equal that obtained through product lines analysis.

The risk associated with places that the firm occupies is not limited to the one calculated above. It can be extended to cover firms which operate internationally. Companies which are multi-national in nature, are constantly facing changes in exchange rate, changes in foreign governments' policies, and changes in the economic conditions of these countries. The risk associated with some of these factors can be quantitatively accounted for in the analysis, but for other factors qualitative measures may be more applicable.

Risk associated with exchange rate (RSKXR). The risk associated with exchange rate is suitable for treating quantitatively. A part of this risk comes through the net income before taxes contributed by the firm's divisions in foreign countries. Another part comes through assets of these divisions physically placed in such countries. Accordingly, we may express this risk by the following equations:

$$RSKXR1 = \sum_{i=1}^{n} X_i CV(XR)_i + \sum_{i=1}^{n} Y_i CV(XR)_i \qquad (57)$$

where n is the number of countries in which the firm has a division, X_i is the proportion of net income before taxes which comes from the division in i-th country, $CV(XR)_i$ is the coefficient of variation of the exchange rate in the i-th country over some period of time, and Y_i is the proportion of the assets placed in the i-th country.

The first term in equation (57) is a weighted average of coefficient of variations of exchange rates of the related countries, whose weights are the relative importance of the contribution of each division in foreign countries to total net income before taxes. The second term is another weighted average of such coefficients whose weights are proportions of assets placed in each country. The more is the company's dependency on the income provided by a foreign division, and the more it has engaged its assets there, the higher is its risk with respect to the exchange rate. From equation (57), it follows that, for a firm which operates only domestically, X_i's and Y_i's are equal to zero, and thus it does not bear any exchange rate risk.

It may be more useful to supplement the above measure with the relative measure of skewness, the standardized third moment about the mean of each of the exchange rates. The negative sign of the moment may indicate depreciative tendency of the changes in the exchange rate, whereas a positive sign may signify appreciative propensity. Since there are no positive correlations between all exchange rates at all times, some component parts of the risk may be cancelled out, if the weights are in appropriate proportions. To express the above points in another way:

$$RSKXR2 = \sum_{i=1}^{n} X_i (a_3)_i + \sum_{i=1}^{n} Y_i (a_3)_i \qquad (58)$$

where $(a_3)_i$ is the standardized third moment of the exchange rate frequency distribution of the i-th country which can be obtained by applying equations (16) and (17).

The risk associated with exchange rate is then a function of equations (57) and (58). Thus:

$$RSKXR = e(RSKXR1, RSKXR2) \qquad (59)$$

Risk associated with age (RSKAGE). The relationship between age and revenue of a firm was discussed previously. This relationship can now be extended to the extreme, the firm's financial distress. It has been an observable fact, as shown by any listing of failed companies, that business failures occur more frequently in a certain age group. Conceivably, this age group differs from industry to industry due to the peculiarities and the complexities involved in and the experience required by each industry. In general, however, younger firms are more apt to fail than older ones. But the relationship is not linear. In fact, over a certain range of the earlier years the function is increasing until it reaches a peak point and then it diminishes. Age, thus, seems to be a relatively important determinant in business failures.

The possibility that a firm faces financial distress is, therefore, partially dependent upon its absolute age as well as its relative age position in a population of failed firms in its industry. To find the risk that a firm bears by being in a certain age group is, then, to find the probability that the age of the firm is the age of failure. To find this probability, take advantage of the Tchebycheff inequality:

$$P(A \leq d) \leq \sigma^2/(E - d)^2 \qquad (60)$$

where A is defined here as failure age variable whose mean value and variance are E and σ^2, respectively, and d is any specified level of age within the range of A.

Relation (60) may now be interpreted in the following way: The probability that the failure age is less than or equal to the age of a specific firm (within the range) is less than or equal to the ratio in the right-hand side of the inequality, where d takes the value of the firm's age. It is seen from (60) that as the age of the firm (d) approaches to the mean value (E), which is the weighted average of all the failed firms in the related industry, the probability that the age of the firm is the age of failure increases. In the extreme, where d = E, this probability would become infinitely large.

This relation does not give the probability for a certain age alone; rather it gives a cumulative probability which includes those of the lower ages. To get the "net" figure, it is necessary to deduct from the probability of each age obtained by (60) the probability of the immediately lower age, as also obtained by (60). It should be noted, however, that all the probability figures obtained by Tchebycheff's inequality are much too conservative. This conservative property increases as d approaches E. The probability figures, nevertheless, provide a useful set of weights to be assigned to firms as risks associated with their relative ages.

Risk associated with expenses (RSKEXP). Just like the case for any other variable whose value may vary over a possible range, expenses are also treated as a random variable. This being so, there exists a possibility that actual expenses differ from what is expected at any point of time. This possibility is what is meant by risk associated with expenses.

Ordinarily, it is expected that an increase in expenses is concurrent with an increase in revenue and a decrease in the former occurs at the same time as a decrease in the latter. But this may not be what actually happens; movements of expenses and revenue may not coincide. Expenses may go up at the same time that revenue declines, or expenses may decline when revenue climbs up. Although not very popular, and although the latter case is highly desirable, detection of the existence of any such relations may shed some additional light on the overall risk exposure of the firm.

One way to find out about such disagreement in the behavior of expenses and revenue is to consider the correlation between them. Correlation between expenses and revenue of a firm can be obtained by utilizing equation (22) in conjunction with equation (21). The combined formula to be used as the determinant of RSKXP1 is presented here:

$$COR(REV,EXP) = \frac{1}{\sigma_{REV}\sigma_{EXP}} \sum_{i=1}^{n} (REV_i - \overline{REV})(EXP_i - \overline{EXP})P_i \qquad (61)$$

where the left-hand term is the correlation coefficient between revenue and expenses, REV_i is the i-th occurring of revenue random variable whose mean and variance are $\overline{REV}$ and σ_{REV}, respectively, EXP_i is the i-th occurring of expenses random variable whose mean and variance are $\overline{EXP}$ and σ_{EXP}, respectively, n is the total number of occurrences, and P_i is the joint probability of the i-th occurrence of both revenue and expenses.

The correlation coefficient takes any value between -1 and +1. Logically, there is more risk attached to the cases which show negative correlation between expenses and revenue than otherwise. Furthermore, the closer is the negative value to minus one, the riskier the firm appears to be, other things being equal.

Existence of negative correlation between expenses and revenue may be due to several factors such as the relations between fixed costs and variable costs, the technology that the firm employs, and the lag between expenses and revenues. In some cases, further exploration of such relations may be rewarding, but this discussion is limited to the seemingly most significant ones.

It is obvious also that the firm bears less risk if the major expense items, i.e., selling, general and administrative expenses are not perfectly positively correlated to each other. Following the portfolio concept, the effect of such correlations on the variance of overall expenses can be found by a treatment similar to that applied to the risk associated with product lines. Thus:

$$RSKXP2 = - \frac{1}{EXP} \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_i \sigma_j \, [1 - COR(i,j)] \quad (62)$$

where n is the total number of major expense items, X_i and X_j are the proportions of total expenses attributed to major items i and j, respectively, σ_i and σ_j are standard devaitions of expenses in major items i and j, and COR(i,j) is correlation between these expenses.

Equation (62) states that the larger is the negative correlation between the major expense items, the larger is the deduction from the overall risk exposure of the firm, and vice versa.

In sum, risk associated with expenses is a function of the above components:

$$RSKXP = f(RSKXP1, RSKXP2) \quad (63)$$

Risk associated with labor (RSKLBR). Closely related to the risk associated with expenses, yet individually of high importance, is the possibility that the actual behavior of the workers of company deviates from what is expected. With an ever-increasing power of the unions, firms are becoming more and more vulnerable to actions taken collectively by workers. Any strike, any wage-increase demand, or any fringe-benefit request has drastic effects on the firm's resources.

A firm whose nature or policy does not warrant full or partial application of automation, relies more on labor. But the higher is the dependency of the firm on labor, the higher is the risk that the firm bears in this regard.

To determine the degree of the dependency, first capitalize the labor expenses by an appropriate discount rate. Then find the ratio of the capitalized labor expenses to total human and non-human assets. The higher is this ratio, the higher is the risk. The reverse is true, of course, for the lower ratio.

Another determinant of the labor risk is the competitive position of the labor. If the laborers are unionized, they are more forceful and impose more risk on the firm than otherwise. On the other hand, if the firm has a monopsony power in employing the labor, its position would have risk-reducing effect. Theoretically, labor and firms could take any position between perfect competition to monopoly or monopsony. In practice, however, one type is more prevalent and that is the unionization of labor.

At any rate, the risk asssociated with labor may be expressed by the following function:

$$\text{RSKLBR} = g(\text{LBRDEP}, \text{LBRCOM}) \tag{64}$$

where LBRDEP is labor dependency ratio and LBRCOM is labor competitive position.

Risk associated with liquid assets. The possibility that liquid assets, i.e., cash and marketable securities, fail to protect the firm from possible adverse deviations of net cash-flow from its expected value has long been considered as risk of technical insolvency.[27] It has also been established that the larger is the proportion of liquid assets, as well as current assets, to total assets, the smaller is the risk.

Also, the general relationship between the risk of technical insolvency and the proportion of current assets which is financed by short-term debt is indisputable. A firm that finances its current assets entirely with equity capital will have less need for liquidity than it would if it financed these assets entirely with short-term borrowings. Thus, for every level of current assets, such firm bears less risk than otherwise.

To these elements should be added the maturity dimension of debts. It is obvious that the larger is the dispersion of the maturity of debt, the less is the risk of technical insolvency. This is so because the long intervals between principal payments of debt give more time to the firm to earn cash or to convert an asset into money in sufficient amounts to meet its obligations.

The maturity data alone, however, may not help very much because it may happen that in spite of seemingly sufficient dispersion of maturities, the firm is still unable to pay the principals of the loans due to their large sizes. It may, therefore, be useful to consider the average size of payments in relation to liquid assets in conjunction with other elements discussed above. To summarize, the above relations may be expressed by the following function:

$$RSKLQ1 = h(LQDAST/TA, CA/TA, CA/CL, MATDSP, SIZPAY/LQDAST) \quad (65)$$

where LQDAST stands for liquid assets, TA for total assets, CA for current assets, CL for current liabilities, MATDSP for maturity dispersion of debts and SIZPAY for average size of principal payments of debts.

As was discussed above, except for the last ratio in equation (65), all other elements have risk-reducing effects as they become larger. Conversely, the larger becomes the size of principal payments in relation to the size of liquid assets, the larger becomes the risk.

It may also be helpful to supplement (65) with some additional information. That is, to provide a series of probability estimates indicating the possibility that any of the elements in (65) will be at or below a disaster level, given their related frequency distribution over a period of time.

For this purpose, calculate the mean value and variance of each of the above-mentioned elements for a company. Also estimate the disaster level of any such element by averaging it over all the recently failed firms in the industry. Then, by utilizating Tchebycheff's inequality, and using the mean, variance, and the estimated disaster level of each element above, there is obtained the related probability. The interpretation of the result obtained in this way is straightforward and is derived from Tchebycheff's inequality. For example, in the case of acid-test ratio,

$$P(LQDAST/TA \leq d_1) \leq \sigma^2/(E - d_1)^2 \quad (66)$$

indicates that, given the mean and variance of the ratio over a period of time, the probability that the ratio falls below the estimated level, d_1, is equal to or less than the right-hand side ratio. A simpler interpretation may be obtained by inverting risk-reducing ratios, but the result would not change. Denoting the probabilities obtained in this way for the elements in (65) by P(1), P(2), P(3), P(4) and P(5), respectively, produces:

$$RSKLQ2 = i\ [P(1),\ P(2),\ P(3),\ P(4),\ P(5)] \quad (67)$$

And the risk associated with liquid assets would be a function of the parts obtained from equations (65) and (67). Thus:

$$RSKLQD = j(RSKLQ1,\ RSKLQ2) \quad (68)$$

Risk associated with securities (RSKSEC). In firms such as financial institutions or securities dealers, where securities other than the marketable ones (which were accounted for in liquid assets discussion) constitute a significant part of total assets of the firm, the risk associated with their collective performance plays an important role in determining the overall risk exposure of the firm.

Finding such risk does not seem difficult after so much progress has been made in the portfolio field. So far, the variance of the return on the securities portfolio has been chosen as a measure of risk. But to obtain it, it is not necessary to resort to the sophisticated model of portfolio variance. Use can be made of the historical data of the return on each security in simple formulas. First calculate the return on the portfolio in each period by the following equation:

$$R(P)_t = \sum_{i=1}^{n} X_i R_i \tag{69}$$

where $R(P)_t$ is return on portfolio at period t, n is total number of securities, X_i is the proportion invested in security i and R_i is return on security i.

Then calculate the mean value of the portfolio return over the period:

$$\overline{R(P)} = \frac{1}{T} \sum_{t=1}^{T} R(P)_t \tag{70}$$

where T is total number of periods under consideration.

The variance is then obtained by:

$$V(P) = \frac{1}{T} \sum_{t=1}^{T} [R(P)_t - R(\overline{P})]^2 \tag{71}$$

And the coefficient of variation of the portfolio by:

$$CV(P) = V(P)/R\overline{(P)} \qquad (72)$$

For present purposes, consideration of the portfolio variance alone, however, does not seem to be sufficient. It seems useful, therefore, to supplement equation (72) with a measure of skewness as well as rate of growth in return on the portfolio.

The relative skewness may be measured by standardized third moment about the mean, as discussed before:

$$a_3 = m_3 / V(P)^{\frac{3}{2}} = 1/V(P)^{\frac{3}{2}} \cdot 1/T \sum_{t=1}^{T} [R(P)_t - R\overline{(P)}]^3 \qquad (73)$$

And the rate of growth of return is obtained by regressing the natural log of the return on portfolio in each period against time, and utilizing the following equation resulted from such regression analysis:

$$\ln R(P)_t = \ln A + rt \qquad (74)$$

where ln R(P) is the natural log of the portfolio return at time t, ln A is y-intercept of the line, and r is the growth rate.

In evaluating the risk of portfolio, the coefficient of determination, R^2, obtained from the regression analysis, is also used along with other factors, because it contains some additional information about the dispersion of the portfolio return.

The final point to be made is that, since portfolio management is a dynamic process in nature, it cannot be assumed that the contents and the size of the portfolio do not change over a period. Thus, the risk associated with the portfolio also changes over a period. In order to evaluate the risk, however, some attempt to treat the problem must be made. One possible way is to take the weighted average of all the factors determining the risk over the last period. The weights, however, would be two dimensional. The first dimension relates to the proportional size of each portfolio variant. The second one relates to the proportion of period over which the portfolio variant was held by the firm. As an example, the weighted average of portfolio variance is obtained by the following equation:

$$\overline{V(P)} = \sum_{i=1}^{n} V(P)_i S_i T_i \qquad (75)$$

where n is total number of the portfolio variants, $V(P)_i$ is the portfolio variance of the i-th variant, S_i is the proportional size of the i-th variant, and T_i is the proportional period (a fraction of 365 days, for example) over which the i-th variant has been held in the last period. Thus, the risk associated with securities is a function of the following factors:

$$RSKSEC = [\overline{CV(P)}, \bar{a}_3, \bar{r}, \bar{R}^2] \qquad (76)$$

where the bars indicate the weighted average of the related factors.

Risk associated with credit extending (RSKCRD). Firms may extend credit to other firms or individuals through either loan or accounts receivable. The incentive is, of course, to increase the sales and profits. Whether in the form of loan, accounts receivable, or any other form, credit extension carries with itself some risk. The risk associated with credit extension is the possibility that the actual result from such action deviates from what is expected.

This risk has two facets: 1) The extension of credit is based upon the expectation that it increases sales and profits, and 2) It is expected that the credit receivers do not default in repaying the entire credits extended to them. Thus, the larger is the deviation of actual results from such expectation, the larger is the related risk.

It is not easy, however, to find out whether the expectation of increasing sales and profits as a result of credit extension is met or not. Nevertheless, a way should be found to estimate its associated risk. One possible way seems to be the determination of the correlation between credits and profits over an appropriate period of time. The correlation coefficient can be calculated by the following equation:

$$\mathrm{COR(NBIT,CRDT)} = \frac{1}{\sigma_{\mathrm{NBIT}}\sigma_{\mathrm{CRDT}}} \sum_{t=1}^{T} (\mathrm{NBIT}_t - \overline{\mathrm{NBIT}})(\mathrm{CRDT}_t - \overline{\mathrm{CRDT}})P_t \quad (77)$$

where the left-hand term denotes the correlation coefficient between net income before interest and taxes and total credit, and on the right T is the number of periods under consideration, NBIT_t is net income before interest and taxes in period t, $\overline{\mathrm{NBIT}}$ and σ_{NBIT} are the mean and standard deviation of the income over T periods, CRDT_t is total credit extended in period t, $\overline{\mathrm{CRDT}}$ and σ_{CRDT} are the mean and standard deviation of credit over T periods, and P_t is the joint probability of the occurrence of NBIT_t and CRDT_t at time t.

A positive correlation coefficient may indicate that actual performance of credit has corresponded to what has been expected. Conversely, a negative coefficient may signal the deviation of actuality from expectations. The larger is the absolute value of the negative correlation, the larger is the deviation, and the larger is the risk undertaken by extending the credit.

If there were not external forces at work when the credit extension were changing, all the changes in net income could be attributed to the variation in credit, but at any point of time numerous factors act and react on net income. In particular, credit is usually eased at the time when the market is weak. It is eased to induce potential buyers to buy and thus to prevent sales from declining. On the other hand, when demand is high, there is no need to ease credit. In sum, however, the correlation coefficient seems to be a useful measure in estimating the risk, but reliance on this factor alone does not seem appropriate. Furthermore, if the correlation is positive, there still is needed some measure to deal with the second facet of the credit risk, the possible default of the credit recipients. Recourse can be made, therefore, to a set of other determinants which may corroborate the correlation coefficient.

The possibility that the credit receivers may default in repaying a part or the entire credit extended to them is a function of several factors. In particular, if a loan is a secured one, the associated risk is usually low, depending upon the nature of the pledge. On the other hand, when it is unsecured, the quality of the accounts and the credit period affect the risk. The better is the quality of accounts, the lower is the risk. But the longer is the credit period, the higher is the risk.

The quality of accounts carried by the firm may be approximated by the bad-debt losses (in percentage of annual sales) and the average collection period. The firms with easy credit policy have both higher bad-debt losses and longer collection periods, whereas the reverse is true for the firms with tighter credit policy.

Another feature of a loan which has an effect on risk is its callability (or withdrawal agreement). This feature gives the lender firm more flexibility in financing its funds requirements and in adjusting their related interest rates.

In summary, the risk associated with credit extending may be expressed by the following function:

$$RSKCRD = 1\ [COR(NBIT,CRDT),\ UNSEC/CRDT,\ BADDBT/SALES,\ AVGCRP,\ ACGCLP,\ CALL/CRDT] \quad (78)$$

where UNSEC stands for unsecured credit, BADDBT is for bad-debt losses, AVGCRP for average credit period, AVGCLP for average collection period, and CALL for total callable credit or loan.

Risk associated with inventories (RSKINV). Inventories provide a very important link between purchase and sale or production and sale. If inventories are not in sufficient and balanced amounts, either customers are hurt and sales deteriorate, or production is hindered and resources lie idle. On the other hand, if inventories are in excess of what are actually needed, there would be a lot of losses involved in carrying costs, opportunity costs, possible obsolescence, price changes, theft, and spoilage.

The risk associated with inventories is, then, the possibility that actual amounts deviate from what is required. This possibility increases as the lead time required to receive delivery of inventory, once an order is placed, is lengthened. Existence of monopoly on the side of the supplier of raw materials also adds to the risk of inventory. Conversely, monopsony power of the firm in buying the supplies has risk-reducing effect.

Despite the constant efforts of theoreticians and practitioners to determine the optimal level and combination of inventories, deviations of actuals from required do occur in practice. Inventory turnover ratio is usually used to measure such deviations. A high turnover ratio may indicate, among other things, that the stock of inventory is low. Similarly, overstock may be shown by a low turnover ratio. But it is difficult to indicate that the turnover ratio of a firm is high or low, unless there is a criterion to be compared with.

One way to meet this purpose is to assume that a large number of similar firms in an industry which are operating successfully have been using a correct level of inventories. Thus, averaging the ratio over such firms gives a criterion for comparison. Therefore, for any similar firm, deviation of its ratio from this amount in the last period is an indication of incorrect level of inventory for the coming period. Thus, a large deviation indicates a high risk, and a small deviation signifies a low risk. To summarize, the risk associated with inventories is a function of the following components:

$$RSKINV = m(DEVINV, SUPCOM, LEADTM) \qquad (79)$$

where DEVINV is the deviation of inventory turnover ratio of the firm from the average ratio of the successful firms in the same period, SUPCOM is the competitive position of the suppliers or the firm (whichever is more important to the firm in determining its risk) with respect to supplies and raw materials, and LEADTM is for lead time.

Risk associated with fixed assets (RSKFA). The risk associated with fixed assets is the possibility that the actual services renderable by these assets deviate from what is expected. The sources of such deviations vary. They may come from technological advancements which render the assets obsolete, or they may be due to untimely physical depreciation. They may also come through inflation. Inflation makes the funds allocated through accounting depreciation insufficient to restore the assets' earning power.

The longer is the life of the fixed assets, the more is the possibility of their obsolescence and untimely depreciation, and thus the higher is the risk of losing their earning power. Conversely, the shorter is the life, the lower is the risk. Also, the higher is the average inflation rate over the period of time under consideration, the higher is the possibility that the accounting depreciation deviates from the economic depreciation[28] of the assets, and thus the higher is the risk associated with the assets.

Another factor which may have some effects on the assets risk is the dispersion of the times at which the economic lives of major items of the fixed assets expire. The smaller is this dispersion, the more difficult it is to replace several useless major items in a short period of time. This may affect the earning power, and thus creates more risk. Conversely, the larger is the dispersion, the lower is the risk. The above discussion can be summarized by the following function:

$$RSKFA = n(LIFE, INFLAT, DISPER) \qquad (80)$$

where life is the economic life of the fixed assets, INFLAT is the average rate of inflation over the period under consideration, and DISPER is the dispersion of times at which the economic lives of major items of the fixed assets are expired.

Risk associated with patents (RSKPAT). The value of patents, goodwill, franchises, and memberships is subject to changes. The possibility that their actual values deviate from what were expected constitutes the associated risk. The deviation may be the result of technological progress, changes in market factors, or changes in law and regulations. At any rate, it seems that the risk is a function of the period that the asset has been in actual use

(PRINUS). In the case of a patent, for example, it seems that the longer it has been in use, the higher is the probability that something similar or even better will be invented. Thus:

$$RSKPAT = o(PRINUS) \qquad (81)$$

Risk associated with debt (RSKDBT). Much has been said about financial leverage and financial risk. For present purposes risk associated with debt is defined as the possibility that the actual ability of the firm to pay its debt obligations, both principal and interest, deviates from what is expected. By increasing the proportion of senior capital in the capital structure, a firm increases its financial leverage. The higher is the level of leverage, the higher is the possibility of its inability to pay, other things being equal, and vice versa.

But, what is the disaster level of leverage below which there is a very high degree of probability that the firm will face financial distress? Theoretically, the answer, which, of course, depends on the peculiarities of the firm and of the industry to which it belongs, has not yet been found. Empirically, it can be assumed that all failed firms in the same industry have had their shareholders' equity capital at or below a disaster level.

Thus, by averaging the proportion of the equity capital in the capital structure of all such firms (preferably of only those firms which are similar to the firm under consideration, with respect to the size and business nature, if their number is large enough) and utilizing the Tchebycheff inequality, it is possible to obtain an approximate probability that the firm's equity capital would be at or below the disaster level.

$$P(EQCAP/TA \leq d) \leq \sigma^2/(E - d)^2 \qquad (82)$$

where EQCAP is equity capital of the firm whose mean and variance over a period of past years is E and σ^2, respectively, TA is total assets, and d is the disaster level.

In addition to the level of debts, their average maturity has also some effects on the related risk. The longer is the maturity, other things being equal, the more time is available to the firm to provide necessary funds, and thus the lower is the risk, and vice versa.

Not only the maturity of debts, but the dispersion of the payment dates also affects the risk. The greater is the dispersion, the less is the burden of payments at any point of time and the easier is the provision of the required funds, other things being equal. Conversely, smaller dispersion imposes more risk on the firm.

If some of the debts have call feature in favor of the firm, it has a risk-reducing effect. Conversely, if the call feature is for the creditors, then the risk of the firm is increased. At any rate, a measure of weighted average of such features, negative for cases in favor of the firm and positive for those against the firm, should be included in the evaluation.

Another factor is accessibility of the firm to the capital market. A firm which has access to the capital market, in addition to being able to obtain cheaper funds, faces less risk, because there is less possibility of being trapped financially.

In summary, the risk associated with debt is a function of the following factors:

$$\text{RSKDBT} = p[P(\text{EQCAP/TA} \leq d), \text{AVGMAT}, \text{PAYDIS}, \text{CALLFF}, \text{CALLAF}, \text{CAPACS}] \tag{83}$$

where AVGMAT is the weighted average debt maturities, PAYDIS is a measure of the dispersion of the payment dates, CALLFF and CALLAF are the weighted average of a measure of call feature for and against the firm, respectively, and CAPACS is the capital market accessibility of the firm.

Risk associated with changes in economic and market factors (RSKECM). The possible relation between a firm's revenue and expenses with the levels of some important economic and market factors in the corresponding periods provides a means of evaluating the profitability of the firm under various conditions, including different phases of business cycles, which seems to be essential in differentiating between firms which operate successfully and those which fail. The possible effects on the performance of the firm in the next period of future changes in economic and market factors, however, constitute an additional risk factor of the firm.

Changes in the economic system are predicted via two general approaches. The first approach utilizes the properties of some economic indicators;[29] the second relies on GNP models.[30] The indicators herald the upturn or downturn of the economy but do not predict the absolute values of the GNP or any other variable. GNP models, on the other hand, predict absolute levels but do not identify turning points.[31]

Since the possibility of a firm's future failure is more likely related to the level of future downward changes in the general business activities than to the mere change itself, it seems more appropriate to take advantage of one or more variables of a GNP model, rather than using the economic indicators. Obviously, no one variable can be equally helpful for all industries in determining the risk associated with changes in the economic factors. Whereas the predicted change in government spending on defense materials, for example, has a great impact on defense-related industries, it may not have any influence on non-defense businesses, and so on.

Therefore, the selected forecasting variable(s) should fit the industry under consideration. For convenience changes in such a variable(s) are referred to as ΔFGBA, i.e., change in a factor(s) of general business activity.

Other determinants of the risk associated with changes in economic and market factors are population trends of the county in which the firm has business (POPTRN), changes in the age composition of the population (ΔCPOP), changes in the competitive forces, such as net change in the number of competitors or predicted change in the company's market share (ΔCOMP), predicted change in environmental factors (ΔENVRN).

In summary, the risk associated with changes in economic and market factors is a function of the following elements:

$$\text{RSKECM} = q(\Delta\text{FGBA}, \text{POPTRN}, \Delta\text{CPOP}, \Delta\text{COMP}, \Delta\text{LAW}, \Delta\text{ENVRN}) \quad (84)$$

In concluding the risk section, it should be added that there may be some additional risk factors which may have some effects on the firm's overall risk exposure, but it seems that all the important ones, at least for present purposes, have been included in the previous discussion.

Mismanagement

In the previous two sections all the factors affecting the first two elements of equation (1), i.e., cash-flow and risk, were discussed at some length. In this section, the third element of the equation, i.e., MISMGT, is discussed, however, rather briefly.

The reason for such treatment of the mismanagement element is the difficulty in generalizing factors which may be common to all firms as having effects in driving a firm out of business, yet do not fall within the domain of the factors discussed so far. To be sure, nevertheless, there may be found some factors which have these characteristics.

Fraud is one of such factors. It is, however, an after-fact phenomenon. Its existence cannot be known until after it is discovered. Embezzlement is another. Manipulation, misconduct, poor books and records, improper control, too rapid expansion, and overcommitment in a particular venture are other such factors which may also be included in the same category.

The effects of these factors varies, of course, in different industries. Whereas embezzlement may be a significant factor in the financial industry, for example, poor books and records may play a vital role in securities industry.

The possible usefulness of including the mismanagement element among the potential determinants of the probability of financial distress of a firm may be better seen through an example from the securities industry. It has been an observable fact in recent years that the probability of financial distress of a brokerage/dealer firm is directly related to fraud or back office problems. In fact, the frequencies with which possible fraud or misconduct and poor books and records were observed in the 94 liquidated firms handled by SIPC during 1971-73 were 55 and 64, respectively.[32]

In summary, mismanagement may be expressed as a function of the following factors:

$$MISMGT = r(FRAUD, EMBEZ, MANIPU, MISCND, IMPCNT, PRBKRC, RAPEXP, OVCOMT) \qquad (85)$$

where FRAUD stands for fraud, EMBEZ for embezzlement, MANIPU for manipulation of the various accounts, MISCND for misconduct, IMPCNT for improper control, PRBKRC for poor books and records, RAPEXP for rapid expansion, and OVCOMT for overcommitment in a particular venture.

Summary and Conclusion

In an effort to present an analytic framework, potentially applicable to all profit-oriented private organizations, for identifying potential determinants of a firm's financial distress, it was hypothesized that the probability of such distress is a function of three elements: net cash-flow per dollar of assets, risk exposure, and mismanagement of the firm.

Because the specific relations between these elements and the probability of the financial distress could not be known, the proposed general relation was expanded to include all possible factors affecting the above-mentioned elements. It was suggested that by exposing this expanded relation to a statistical analysis, it may be possible to obtain a specific relation between some of the factors which are more important than others in determining the probability of the financial distress of a firm in a specific industry.

In identifying the factors which affect a firm's net cash-flow per dollar of assets, the revenue side and the expense side of the firm were treated separately. The effects of all internal as well as external factors on both revenue and expenses were then examined individually. The result was the emergence of a set of general relations numbered (4) through (11).

The treatment of the risk element was organized by first adopting a definition of risk. It was defined as the possibility that actual outcome of a random variable will deviate from that which is expected. Then, variance or standard deviation of a single variable, semi-variance, third moment, rate of growth, confidence level, covariance, correlation coefficient, variance of a group of variables, and the probability of disaster level were discussed as possible measures which could individually, or in combination, determine the extent of a risk associated with any factor. Finally, risk factors which collectively determine the risk exposure of the firm were examined in detail.

Included were risks associated with cash-flow, product lines, places occupied, age of the firm, expenses, liquid assets, credits, inventories, fixed assets, patents, borrowing, and changes in economic and market factors.

The last section discussed briefly the suitability of including in our analysis some other factors which do not fall in the domain of the other two elements discussed above but may have some impact on the possibility of a firm's financial distress. This resulted in the MISMGT relation numbered (85).

Now, by replacing the subordinate functions into the secondary ones, and the secondary ones into the original one, there is obtained the expanded form of equation (1) that contains all of the factors which possibly have effect on the probability of a firm's financial distress. It is this last function which is subject to an empirical test to be conducted by using multivariate discriminant analysis technique.

NOTES

1. The relative importance of cash-flow over income has been emphasized in many financial texts. See, for example, D.E. Peterson, A Quantitative Framework for Financial Management, Homewood, Ill.: Richard D. Irwin, Inc., 1969, P. 335.

2. Donaldson suggests the determinants of net cash-flow as being sales collections, other cash receipts, payroll expenditures, raw material expenditures, and non-discretionary cash experditures. See G. Donaldson, Corporate Debt Capacity, Boston: Division of Research, Harvard Business School, 1961; and G. Donaldson, "Strategy for Financial Emergencies," Harvard Business Review, Vol. 47 (1969), pp. 67-79.

3. For a theoretical discussion of the optimal levels of investment in various assets, see J. C. Van Horne, Financial Management and Policy, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., Second Edition, 1971, pp. 383-403.

4. See D. Durand, "The Cost of Debt and Equity Funds for Business," reprinted in The Management of Corporate Capital, ed. E. Solomon, New York: The Free Press, 1959, pp. 91-116; also, F. Modigliani and M.H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review, Vol. 48 (1958), pp. 261-97.

5. For a discussion on the optimal composition of short-term financing, see J.C. Van Horne, op. cit., pp. 533-537.

6. P.B. Gove and the Merriam Webster Editorial Staff, ed., Webster Third New International Dictionary, Springfield, Mass: G. and C. Merriam Co., 1961. p. 1961.

7. H.S. Sloan and A.J. Zurcher, A Dictionary of Economics, 4th ed., New York: Barnes & Noble, Inc., 1965, p. 289.

8. D. Greenwald and Associates, The McGraw-Hill Dictionary of Modern Economics, New York: McGraw-Hill Book Company, 1965, p. 446.

9. R.E. Machol and E.M. Lerner, "Risk, Ruin and Investment Analysis," Journal of Financial and Quantitative Analysis, Vol. 4 (1969), pp. 473-92.

10. For more information, see A. Robicheck, "Risk and the Value of Securities," Journal of Financial and Quantitative Analysis, Vol. 4 (1969), pp. 513-38.

11. A "random variable" is defined as a variable that takes on certain values, each corresponding to a particular outcome of some uncertain situation. See K.V. Smith, Portfolio Management: Theoretical and Empirical Studies of Portfolio Decision Making, New York: Holt, Rinehart and Winston, Inc., 1971, p. 69n.

12. J. Tobin, "Liquidity Preference as Behavior Towards Risk," Review of Economic Studies, Vol. 25 (1958), p. 72.

13. H.M. Markowitz, Portfolio Selection: Efficient Diversification of Investments, New York: John Wiley and Sons, Inc., 1958, pp. 287-297.

14. Ibid., Chapters IV and IX.

15. Ibid., p. 191.

16. For more information about the measures of relative skewness and kurtosis including peakedness and flat-toppedness, see D.J. Cowden, Statistical Methods in Quality Control, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1957, Chapter 2.

17. F.D. Arditti, "Risk and the Required Return on Equity," Journal of Finance, Vol. 22 (1967), pp. 19-36.

18. The other risk variables were coefficient of correlation between the returns from a single stock and all other available stocks, dividend to earnings ratio, and debt to equity ratio.

19. For further information, see A.C. Chiang, Fundamental Methods of Mathematical Economics, New York: McGraw-Hill Book Co., 1967, pp. 275-278.

20. For further interpretation of R^2, see, for example, W. Mendenhall, Introduction to Linear Models and

the Design and Analysis of Experiments, Belmont, California: Duxbury Press, A Division of Wadsworth Publishing Co., Inc., 1968, pp. 198-200.

21. W.J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," Management Science, Vol. 10 (1963), pp. 174-182.

22. W.F. Sharpe, "A Simplified Model for Portfolio Analysis," Management Science, Vol. 9 (1963), pp. 277-293.

23. A.D. Roy, "Safety First and the Holding of Assets," Econometrica, Vol. 20 (1952), pp. 431-449.

24. Donaldson, op. cit.

25. According to Jacoby, during 1968 alone more than 4,200 companies disappeared by mergers (including combinations and acquisitions) involving an estimated 43-billions worth of securities. N.H. Jacoby, "The Conglomerate Corporation," Financial Analysts Journal, Vol. 26, No. 3 (1970), pp. 35-48.

26. Other reasons for conglomeration which are mentioned frequently are growth, management acquisition, tax and financing advantages, and operating economies. Some writers believe that operating economies are the only justification for conglomeration, because diversification can be made by shareholders themselves. See, for example, D.C. Mueller, "A Theory of Conglomerate Merger." Quarterly Journal of Economics, Vol. 83 (1969), pp. 652-653.

27. As opposed to legal insolvency which is negative net worth and occurs whenever liabilities exceed the assets of a firm, technical insolvency occurs whenever a firm is not able to meet its cash obligations. For further information on technical insolvency, see J.E. Walter, "Determination of Technical Solvency," Journal of Business, Vol. 30 (1975), pp. 30-43.

28. Economic depreciation has been defined as the amount of investment needed to keep the earning power of an asset the same. For additional discussion on the effects of inflation on the assets, see E.M. Lerner and W.T. Carleton, A Theory of Financial Analysis, New York: Harcourt Brace Javanovich, Inc., 1966. Chapter 4.

29. For one of the popular and widely used list of economic indicators, refer to G.H. Moore and J. Shiskin, Indicators of Business Expansions and Contractions, National Bureau of Economic Research Occasional Paper No. 103, New York: Columbia University Press, 1967.

30. See, for example, J. Duesenberry, O. Ecksein, and G. Fromm, "A Simulation of the United States Economy in Recession," Econometrica, Vol. 28 (1960), pp. 749-809.

31. For a discussion on the differences between these two general approaches to economic forecasting, see R.A. Gordon, "Alternative Approaches to Forecasting: The Recent Work of the National Bureau," The Review of Economics and Statistics, Vol. 44 (1962), pp. 284-291.

32. Securities Investor Protection Corporation (SIPC), Third Annual Report 1973, p. 24.

CHAPTER III

MULTIVARIATE DISCRIMINANT ANALYSIS

Introduction

Multivariate discriminant analysis (MDA) is a statistical technique which can be used to describe the differences between two or more groups and to classify an individual into one of several populations based on a set of characteristics which are observable in the members of the groups. More formally, suppose that there are k groups or populations, π_1, π_2, ..., π_k, with a mean vector of p measurements, $\mu'_i = (\mu_{i1}, \mu_{i2}, \ldots \mu_{ip})$, for i = 1, 2, ..., k. The problem to be solved by MDA is to measure the distance between these groups and to assign an individual with a vector of measurements $x' = (x_1, x_2, \ldots, x_p)$ to one of these k populations to which it has the closest similarity with respect to such measurements.

In practice, however, the mean vector of populations is not known and should be estimated by samples taken from such populations. In fact, the important characteristics of the populations are unknown, as well. The MDA technique can be used in such a way that, by using a sample from each population, the significant features of the populations are extracted first and, then, a classification rule is established on the basis of these features. The procedure of MDA may be better illustrated by using an example given by Duda and Hart,[1] although in a "pattern classification" context.

Consider a lumber mill which wants to automate the process of sorting finished lumber according to species of tree. A system to perform this task might have the form shown in Figure 1. The camera passes the data to the feature extractor. The latter selects and measures some of these features and passes them to the classifier. The classifier then evaluates the presented evidence and mades a decision about the lumber type.

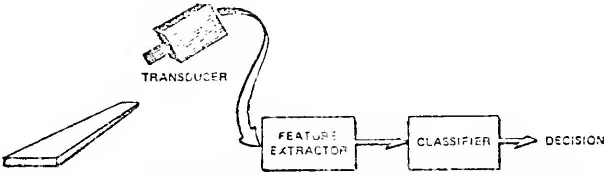

FIGURE 1. A PATTERN CLASSIFICATION SYSTEM.

Source: Duda and Hart's book (see note 1). Reproduced by permission.

In applying the MDA, the work of a researcher may be likened to that of the transducer in the above system. The researcher considers all of the relevant features and properties which may contribute to distinguishing between groups. He then makes them available to the MDA. A part of the analysis performs the task of the feature extractor. This is done by a stepwise procedure in which the "best" variables are selected. Based on the selected variables one or more discriminant functions are constructed and a classification rule is established to act as the classifier in the above system. These features and rules are then used to classify a new observation into an appropriate group.

The highly desirable features of MDA have rapidly expanded its application from its initial use in the fields of anthropology and plant taxonomy to such other diverse fields as biology, medicine, education, psychology, and economics. Yet, all its potentialities have not been exhausted, and it is expected that its application will extend to a wide variety of new fields in the near future.

It is congruent with these features that the present study attempts to apply the MDA technique to the problem of identifying the factors affecting the financial distress of a securities brokerage/dealer firm and predicting its performance, in terms of potential bankruptcy or nonbankruptcy, for a sufficiently long period of time in advance.

The purpose of this chapter is to present an overall picture of the MDA technique in application in a summary form. The mathematical formulations are mentioned without trying to show their derivations, and the related theories are discussed only up to a point where they have direct bearing on the understanding of the application. A section on the historical development, however, provides sufficient references for a reader who wishes a deeper knowledge on the subject. A discussion of the general assumptions gives an idea about the conditions under which the results are more reliable. Relation to regression analysis, relative importance of variables, reliability tests, and the available computer programs are contingent topics to be discussed. The chapter concludes after a brief survey on the application of MDA in the financial area.

General Assumptions

The MDA technique is based on the following three general assumptions:

(1) The populations are identifiable and can be distinguished from each other. In fact, some authors require that the populations be mutually exclusive and exhaustive. Examples of such populations are: good and bad credit risks; successful and unsuccessful salesmen; bankrupt and nonbankrupt firms; diabetic and nondiabetic persons; and male and female groups. The larger is the distance between two populations[2] or the smaller is the overlap between the probability distributions of the populations with respect to some common features, the more suitable is the application of the MDA technique and the smaller is the probability of misclassification of an individual, and vice versa.

(2) Each member of a population can be characterized by a vector of measurable attributes $x' = (x_1, x_2, \ldots, x_p)$, where p is the number of attributes. Also, the subject populations are differentiated from each other by the vector of the means of such attributes in each population, $\mu'_i = (\mu_{i1}, \mu_{i2}, \ldots, \mu_{ip})$, for $i = 1, 2, \ldots, k$.

(3) The attributes x_j, $j = 1, 2, \ldots, p$, are distributed according to a continuous multivariate normal law in each of the populations.[3] In practice, however, measurement is too primitive to yield a continuous scale. Furthermore, there are many situations in which the variables are dichotomous and the data may take the form of the existence versus nonexistence of a condition. Under certain conditions, i.e., equality of variance-covariance matrices and largeness of the discriminating variables, the violation of this assumption does not invalidate the MDA technique. By means of computer programs, Gilbert, for example, concluded that ". . .as the number of variables increases, Fisher's linear discriminant function, which utilizes only bivariate marginals, should remain fairly stable. . . ."[4]

Many researchers have used Fisher's LDF in situations where non-normal and qualitative variables were present.[5] They have justified its usage by the fact that Fisher has shown that this function maximizes the between sum of squares relative to the within sum of squares, regardless of the distribution of x.[6]

Historical Development

Many writers attribute the development of the multivariate discriminant analysis to R.A. Fisher, who formally published the concept and the application of the "linear discriminant function" in 1936.[7] The idea of discriminating between multivariate populations, however, could be traced back to 1920. In this year, Karl Pearson was busy in developing a "measure of distance," called C^2, between two populations in an analysis of anthropometric data.[8] But the first work on his coefficient of racial likeness was published by Miss Tildesly[9] in 1921. During the period 1921-1936 several noticeable progresses were made in this field: Mahalanobis[10] developed his measure of distance, called D^2, Hotelling[11] generalized "student's t," called T^2, and Neyman and E.S. Pearson[12] presented the theory of testing hypothesis.

According to Girshick,[13] the development of discriminatory analysis started with a Pearsonian stage, followed by a Fisherian and a Neyman-Pearson stages, and led to a Waldian stage. Up to about 1950, the field benefited from the influential works of only a few writers, whose number were not comparable to those in the later periods. Gradually, however, this branch attracted many researchers from different disciplines. And their contributions flooded the literature under a variety of names such as prediction, allocation, identification, selection, classification, pattern recognition, and discrimination.

In fact, attraction of the field soon passed the border of individual researchers and captured the attention of many scientific organizations. NATO Science Committee, for example, has been supporting many research works in this area for several years and two of the following-referenced papers[14] are among another 16 which comprised the proceedings of the NATO Advanced Study Institute on Discriminant Analysis and Applications,[15] held in Athens, Greece, during June 1972.

The limitation of space in the present study precludes making individual references to such early and later works. A practical alternative, however, is to name the surveys and bibliographical works which have been made in this regard, so far. The first survey of discriminatory analysis was made by Hodges[16] in 1950. It was sponsored by the USAF School of Aviation Medicine and distributed in a mimeograph form. In 1962, Posten prepared a bibliography on classification, discrimination, generalized distance, and related topics.[17] This work was also mimeographed. A comprehensive printed bibliography, although in the general field of the multivariate statistical analysis, became available in 1972. It was prepared by Anderson, Das Gupta and Styan.[18]

More recently, Das Gupta has presented a comprehensive review on classification.[19] He has, however, concentrated on major theoretical papers and deliberately excluded many applied papers and computer programs. Included in his review is a fairly extensive bibliography arranged by subject matter and then chronologically within subject matter. The last work in this respect is that of Cacoullos and Styan.[20] It is an almost exhaustive bibliography of discriminant analysis for the time period covered. It includes a list of 547 research papers and 26 books which were published up to and including 1971 and 1972, respectively.

Finally, mention should be made of the bibliography prepared by Toussaint.[21] It is, however, on estimation of misclassification.

Test of Equality of Dispersion Matrices

Before any attempt is initiated to compute a discriminant function, having knowledge about the equality of dispersion matrices of the sampled group is the first order of importance. If these matrices are equal, the proper discriminant function is a linear one. Otherwise, quadratic classification procedure should be used.

The effect of the dispersion heterogeneity, however, differs from case to case. In general, the extent of the bias introduced into the result by ignoring the inequality depends upon the degree of the heterogeneity of the dispersion matrices, the number of variables in the function, the number of groups, and the size of the sample. Under the title "The Robustness of Hotelling's T^2," and by using Monte Carlo methods for generating sample data, Holloway and Dunn[22] studied some of these factors in the two-group case. They presented their results in several graphs and showed that in the presence of dispersion inequality, the null hypothesis stating that two vector means are equal is accepted more frequently by the significant test.[23] As a matter of illustrating the point, some of their interesting graphs are reproduced here. Their notations have been defined as the following: p is number of variates, N_1 and N_2 are sample sizes of groups 1 and 2, $r = N_1/(N_1 + N_2)$, Σ_1 and Σ_2 are covariance matrices of the two populations, and d is equal eigenvalues of the matrix $\Sigma_2\Sigma_1^{-1}$.

Figure 2 shows that, for each number of variates, as the sample size decreases, the actual level of significance (α) increases. This effect is more pronounced when the number of variates increases from $p = 1$ to $p = 10$. In other words, as the number of variates increases, the sample necessary for the test to be relatively robust

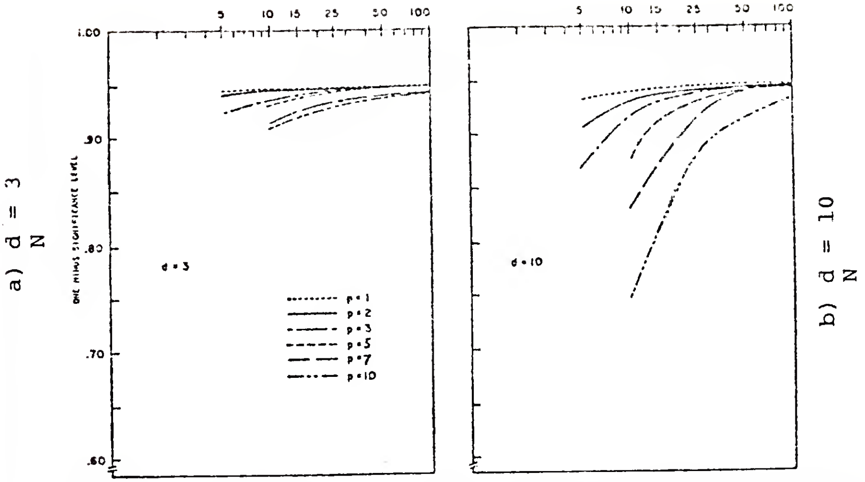

FIGURE 2. HOTELLING'S T^2; ONE MINUS SIGNIFICANCE LEVEL AS A FUNCTION OF SAMPLE SIZE

$1 - \alpha = 0.95$

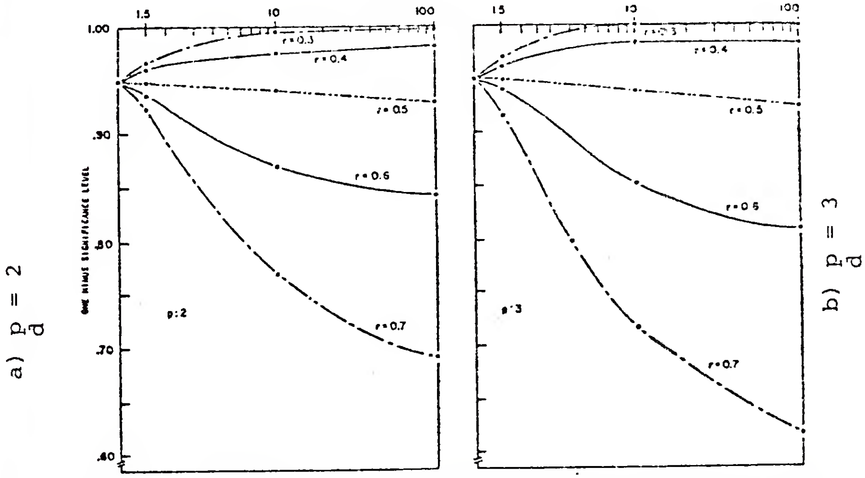

FIGURE 3. HOTELLING'S T^2; ONE MINUS SIGNIFICANCE LEVEL AS A FUNCTION OF d FOR SEVERAL VALUES OF

$r = N_1/(N_1+N_2)$

$1 - \alpha = 0.95;\ N_1+N_2 = 50$

Source of both figures: Holloway and Dunn's article (see note 22). Reproduced by permission.

becomes large. The effect is still more pronounced when d increases from 3 to 10.

Figure 3 shows that, for each ratio r, as the value of d increases, the actual level of significance (α) increases. For a particular value d, the significance level is seen to increase as r increases from 0 to 1. This effect is more pronounced when the number of variates increases, say, from $p = 2$ to $p = 3$.

Similar studies have been made by Hopkins and Clay,[24] and Ito and Schull.[25] Hopkins and Clay concluded that the distribution of T^2 for $p = 2$ with $N_1 = N_2 \geq 10$ is rather robust in respect to variance inequality, but that this robustness does not extend to unequal sample sizes. The results of the Ito and Schull study indicates that for $p = 1, 2, 3$, and 4, with very large samples of nearly equal size, inequality of covariance matrices does not appreciably affect the level of significance or the power of T^2 test as long as the characteristic roots of $\Sigma_2 \Sigma_1^{-1}$ remain within the range (0.5, 2).

Although these last studies extenuate the concern about the inequality of dispersion matrices, they do not mitigate the necessity of conducting the related test. This is so, because the conditions set forth for the ineffectiveness of the heterogeneity are very restrictive and are less likely to be met in practice.

Several approaches have been presented[26] for testing the equality of dispersion matrices, but the one which is more frequently cited is the approach taken by Box.[27] This approach tests the H_1 hypothesis, i.e., $H_1 : \Sigma_1 = \Sigma_2 = \ldots = \Sigma_k$, by an F statistic. To carry out the test the following quantities are calculated first:

$$A_1 = \frac{2p^2 + 3p - 1}{6(k-1)(p+1)} \left(\sum_k \frac{1}{N_k} - \frac{1}{N}\right), \text{ and} \qquad (1)$$

$$A_2 = \frac{(p-1)(p+2)}{6(k-1)} \left(\sum_k \frac{1}{N_k^2} - \frac{1}{N^2}\right), \qquad (2)$$

where p = number of variables, k = number of groups, N_k = number of observations in the k-th sample, and $N = \sum_k N_k$. If $A_2 - A_1^2 \geq 0$, the test statistic, with f_1 and f_2 degrees of freedom, is

$$F^{f_1}_{f_2} = \frac{M}{b}, \qquad (3)$$

where $f_1 = \frac{1}{2}(k-1)\ p\ (p+1)$, (4)

$$f_2 = (f_1+2)/(A_2-A_1^2), \qquad (5)$$

$$b = f_1/(1-A_1-f_1/f_2), \qquad (6)$$

$$M = N\ln|S_{ij}| - \sum_k (N_k \ln|S_{ijk}|), \qquad (7)$$

Where S_{ijk} is the usual unbiased estimate of the variance or covariance between the i-th and j-th variable in the k-th sample, and

$$S_{ij} = (\sum_k S_{ijk})/N. \tag{8}$$

If $A_2 - A_1^2 < 0$, $f_2 = (f_1+2)/(A_1^2 - A_2)$, (9)

and the test statistic is

$$F_{f_2}^{f_1} = f_2 M/[f_1(b-M)] \tag{10}$$

Mathematical Formulations for Sample Use

Although the procedure of classification of individuals into appropriate groups is more easily derived for the cases in which the parameters of multivariate normal populations (i.e., mean vectors and dispersion matrices) are known, only the cases in which the parameters are not known will be considered here. Such a choice arises from two considerations: (1) the purpose of this study is not to delve into the mathematics of the MDA technique;[28] and (2) in most applications of the MDA theory, the parameters are not known and are inferred from samples.

In presenting the subject, the matter related to the two-group case is introduced first, then the k-group case is discussed. In each case, also, the situations of equal and unequal dispersion matrices are considered separately.

The Two-Group Case

Equal disperson matrices. This has been by far the most prevalent case in research works, either due to the nature of data or justified (and, at times, unjustified) assumptions made by researchers.

Test of significance. Before any attempt is made for fitting a discriminant function, the following hypothesis should be tested to ascertain the equality of the mean vectors of the two populations:

$$H_2 : \mu_1 = \mu_2 \qquad (11)$$

If the hypothesis is not rejected, it may indicate that the means of the two populations are so close to each other and the overlap of their related frequency distribution curves is so large that no practical difference can be found between the two groups. In such a case the predicting power of any function to classify an individual into one of these groups will be too weak to be useful. Thus, only if the hypothesis is rejected, continuation of the analysis makes sense.

Several test statistics have been developed for testing the H_2 hypothesis.[29] To name a few, Hotelling's

T^2, Mahalanobis' D^2, and Wilks' Λ are some of such tests which have been shown to be interrelated.[30] Hotelling's T^2 statistic, in the way cited by Anderson,[31] sets the critical region in the following form:

$$T^2 \geq \frac{(N_1+N_2-2)p}{N_1+N_2-p-1} \cdot F^{p}_{N_1+N_2-p-1}(\alpha) \qquad (12)$$

with significant level α, where

$$T^2 = \frac{N_1 N_2}{N_1+N_2} \; (\bar{X}_1-\bar{X}_2)' \; S^{-1} \; (\bar{X}-\bar{X}_2), \qquad (13)$$

where $(\bar{X}_1-\bar{X}_2)$ is the group mean-difference vector of p variables and S is the pooled within-group variance matrix whose element is:

$$S_{ij} = \frac{1}{N_1+N_2-2} \sum_{K=1}^{2} \sum_{m=1}^{N_k} (X_{kim}-\bar{X}_{ki}) \, (X_{kjm}-\bar{X}_{kj}), \qquad (14)$$

where X_{kim} is the m-th observation ($m = 1, 2, \ldots N_k$) for the i-th variable ($i, j = 1, 2 \ldots, p$) in the k-th sample ($k = 1, 2$) from the k-th normal population with (μ_k and Σ) parameters, and $\bar{X}_{ki}$ is the k-th group mean of the i-th variable.

The discriminant function. Given that H_1 has not, but H_2 has been rejected, the appropriate discriminant function is a linear one in the following form:

$$Z = \sum_{i=1}^{p} B_i X_i \qquad (15)$$

There are several approaches to assign values to B's coefficients, based on the information contained in samples. Ladd, however, has shown that most of the alternatives are proportional to each other.[32] As the first approach, he derives B vector in such a way that the between-group variance is maximized relative to within-group variance. In other words, he determines B's to maximize the following function:

$$G = B'dd'B/B'SB, \qquad (16)$$

where $d = (d_1, d_2, \ldots, d_p)'$ = column vector of group-mean-differences $(\overline{X}_1 - \overline{X}_2)$. The solution to the above function is the vector of

$$B = \frac{\alpha}{\lambda} S^{-1}d, \qquad (17)$$

where $\frac{\alpha}{\lambda}$ is a constant: α is a scalar resulting from $d'B$, and λ is a Lagrange multiplier.

Criterion for classification. After the discriminant function is constructed in one way or the other, with respect to the B's various alternatives, one may wish to use it to classify one or more observations as belonging to one or the other population. For this purpose, Anderson proposes the following classification rule:[33] Assign the observation to group 1, if the following inequality holds and to group 2, if its strict reverse is true:

$$X' S-1 \; (\overline{X}_1-\overline{X}_2) \geq \frac{1}{2} \; (\overline{X}_1+\overline{X}_2)' \; S^{-1} \; (\overline{X}_1+\overline{X}_2) + \ln L \qquad (18)$$

where $L = q_2 \; C(1|2)/ \; q_1 C(2|1)$, (19)

and q_1 and q_2 are a priori probabilities of drawing an observation from population π_1 and π_2, respectively, $C(1|2)$ is the cost of misclassifying an observation from π_2 as from π_1, and $C(2|1)$ an observation from π_1 as from π_2. The left-hand term of the above inequality is the subject linear discriminant function.[34] Thus, if we let

$$B = S^{-1} \; (\overline{X}_1-\overline{X}_2) \qquad (20)$$

Then, the above inequality becomes:

$$X' B \geq \frac{1}{2} \; (\overline{X}_1+\overline{X}_2)' B+\ln L \qquad (21)$$

In the case that $q_1 = q_2$ and $C(1|2) = C(2|1)$, the last term drops. The rule then becomes: assign the observation to group 1 if,

$$X' B \geq \frac{1}{2} \; (\overline{X}_1+\overline{X}_2)' B \qquad (22)$$

and to group 2, if

$$X'B < \frac{1}{2}\ (\bar{X}_1+\bar{X}_2)'\ B \qquad (23)$$

Unequal disperson matrices. So far, the discussion has revolved about the two-group case with equal dispersion matrices. If the matrices are not equal, however, the previous discussion should be revised in the following way:

Test of significance. To test H_2 hypothesis, Anderson[35] suggests two T^2-tests, one for the case when $N_1 = N_2 = N$ and the other for the case when $N_1 < N_2$. When $N_1 = N_2 = N$,

$$T^2 = N\ (\bar{X}_1-\bar{X}_2)'\ S^{-1}\ (\bar{X}_1-\bar{X}_2), \qquad (24)$$

$$\text{where } S = \frac{1}{N-1} \sum_{\alpha=1}^{N} (X_{1\alpha}-X_{2\alpha}-\bar{X}_1+\bar{X}_2)\ (X_{1\alpha}-X_{2\alpha}-\bar{X}_1+\bar{X}_2)' \qquad (25)$$

This test has T^2-distribution with N-1 degrees of freedom. When $N_1 < N_2$,

$$T^2 = N_1\ (\bar{X}_1-\bar{X}_2)'\ S-1\ (\bar{X}_1-\bar{X}_2), \quad \text{where} \qquad (26)$$

$$S = \frac{1}{N_1-1} \sum_{\alpha=1}^{N_1} [X_{1\alpha}-\bar{X}_1-\sqrt{\frac{N_1}{N_2}}\ (X_{2\alpha}-\frac{1}{N_1} \sum_{\beta=1}^{N_1} X_{2\beta})] \cdot$$

$$[X_{1\alpha}-\bar{X}_1-\sqrt{\frac{N_1}{N_2}}\ (X_{2\alpha}-\frac{1}{N_1} \sum_{\beta=1}^{N_1} X_{2\beta})]', \text{ for } \alpha \neq \beta \qquad (27)$$

This test also has T^2-distribution with N_1-1 degrees of freedom.

The function. In the case of unequal dispersion matrices, the discriminant function would have the quadratic form.[36]

$$Z = \frac{1}{2} \ln\left|\frac{S_2}{S_1}\right| - \frac{1}{2} \left[(X-\bar{X}_1)' S_1^{-1} (X-\bar{X}_1) - (X-\bar{X}_2)' S_2^{-1} (X-\bar{X}_2)\right] \qquad (28)$$

where S_1 and S_2 are the sample dispersion matrices, whose elements are:

$$(S_1)_{ij} = \frac{1}{N_1-1} \sum_{\alpha=1}^{N_1} (X_{1i\alpha}-\bar{X}_{1i})(X_{ij\alpha}-\bar{X}_{ij}) \qquad (29)$$

$$(S_2)_{ij} = \frac{1}{N_2-1} \sum_{\alpha=1}^{N_2} (X_{2i\alpha}-\bar{X}_{2i})(X_{2j\alpha}-\bar{X}_{2j}) \qquad (30)$$

The quadratic function contains the linear terms and the quadratic terms. For a three-variable case, the previous general function (28) results in the following form:

$$Z = \alpha_{10}X_1 + \alpha_{20}X_2 + \alpha_{30}X_3 + \alpha_{11}X_1^{\;2} + \alpha_{21}X_1X_2$$
$$+ \alpha_{22}X_2^2 + \alpha_{31}X_1X_3 + \alpha_{32}X_2X_3 + \alpha_{33}X_3^2, \qquad (31)$$

Which can be cast into the following form, for a mnemonic purpose:

	Linear Terms	Var. 1	Var. 2	Var. 3
Var. 1	α_{10}	α_{11}		
Var. 2	α_{20}	α_{21}	α_{22}	
Var. 3	α_{30}	α_{31}	α_{32}	α_{33}

Criterion for classification. The classification rule is obtained directly from the discriminant function. Let

$$\chi_1^2 = (X-\bar{X}_1)' \; S_1^{-1} \; (X-\bar{X}_1) \tag{32}$$

$$\chi_2^2 = (X-\bar{X}_2)' \; S_2^{-1} \; (X-\bar{X}_2) \tag{33}$$

Equation (28) then becomes:

$$Z = \frac{1}{2} \ln\left|\frac{S_2}{S_1}\right| - \frac{1}{2} (\chi_1^2-\chi_2^2) \tag{34}$$

And the rule for classification is: Assign the observation to group 1, if

$$\chi_1^2 \leq \chi_2^2 + \ln\left|\frac{S_2}{S_1}\right| - 2\ln L, \tag{35}$$

and to group 2, otherwise. L is the same as defined in (19). Again, if the *a priori* probabilities of the two populations and their costs of misclassification are equal, the last term on the right-hand side of the above inequality drops out.

The k-Group Case

Equal dispersion matrices. When the number of groups exceeds two, the elegance that the Fisher's solution enjoys in the two-group case disappears. The procedure of discriminating among several populations is, however, similar to that of the two-population case. As before, the sample space is divided by discriminant functions into k mutually exclusive regions. In this case, the number of discriminant functions is more than one. If the number of variables, p, is greater than the number of populations minus one, k-1, the number of discriminant functions required would be k-1. Otherwise, it would be p.[37]

Test of significance. The following H_2 hypothesis may be tested by two methods. The first one is the Bartlett V, which is the first approximation. The second one is that suggested by Rao, which is a better approximation.[38]

$$H_2 : \mu_1 = \mu_2 = \ldots = \mu_k \tag{36}$$

The Bartlett V statistic is

$$V = -m \ln\Lambda = \chi^2_p \ (k-1) \tag{37}$$

where $m = N-1-(p+k)/2,$ (38)

and $\Lambda = |W|/|T|$ (39)

An element of W and an element of T are:

$$(W)_{ij} = \sum_{k=1}^{K} \sum_{n=1}^{N_k} (X_{kin}-\overline{X}_{ki})\,(X_{kjn}-\overline{X}_{kj}), \qquad (40)$$

$$(T)_{ij} = \sum_{k=1}^{K} \sum_{n=1}^{N_k} (X_{kim}-\overline{X}_{i})\,(X_{kjn}-\overline{X}_{j}), \qquad (41)$$

The chi-square in (37) has p(k-1) degrees of freedom. The Rao test is:

$$F^{2r}_{ms+2\lambda} = \frac{1-y}{y} \cdot \frac{ms+2\lambda}{2r}, \qquad (42)$$

where

$$y = \Lambda^{\frac{1}{s}} \qquad (43)$$

$$s = \left(\frac{p^2(k-1)^2-4}{p^2+(k-1)^2-5}\right)^{\frac{1}{2}} \qquad (44)$$

$$\lambda = [p(k-1)-2]/4, \text{ and} \qquad (45)$$

$$r = p(k-1)/2 \qquad (46)$$

The chi-square test may also be used to test the significance of the j-th discriminating function, given that the previous (j-1) functions are significant. For this purpose, equation (37) can be written in the following form:

$$V = [N-1-(p+k)/2] \sum_{j=1}^{r} \ln\,(1+\lambda_j) = \chi^2\, p(k-1), \qquad (47)$$

where the relation between Λ and λ_j is as the following (λ_j's are ordered such that $\lambda_1 > \lambda_2 > \ldots > \lambda_r$):

$$\Lambda = \frac{|W|}{|T|} = \prod_{j=1}^{r} \frac{1}{1+\lambda_j} \tag{48}$$

The successive terms ln $(1+\lambda_j)$ are statistically independent. Therefore, each of the additive components of V is distributed approximately as chi-square with p+k-2j degrees of freedom.[39] That is,

$$V_j = [N-1-(p+k)/2] \ln (1+\lambda_j) = \chi^2_{p+k-2j} \tag{49}$$

By cumulatively subtracting $V_1, V_2, \ldots, V_r$ from V, we get, each time, a remainder which has also a chi-square distribution with p(k-1)-(p+k-2j) degrees of freedom and is an appropriate statistic for testing the significance of the residual discrimination. If the residual is significant, given that the first j-1 functions are also significant; it would indicate that the j-th function is significant. If the residual is not significant at the specified level, it may be an indication that only the first j-1 functions have discriminatory powers and the remaining functions may be disregarded. This test, of course, is not an exact one and should be applied cautiously.

The functions. Under the assumption that q_i and q_j, *a priori* probabilities of drawing an observation from populations π_i with density $P_i(X)$ and π_j with density $P_j(X)$, respectively, are known, the regions of classification R_i and R_j are defined by a set of discriminatory functions which minimize the expected cost of misclassification. These functions are linear and are constructed by the likelihood ratio:[40]

$$U_{ij}(X) = \log \frac{P_i(X)}{P_j(X)} = [X-\frac{1}{2}\,(\bar{X}_i+\bar{X}_j)]\;S^{-1}\;(\bar{X}_i-\bar{X}_j), \qquad (50)$$

where S is defined by

$$(\sum_{i=1}^{k} N_i-k)S = \sum_{i=1}^{k}\;\sum_{\alpha=1}^{N_i}\;(X_{i\alpha}-\bar{X}_i)\;(X_{i\alpha}-\bar{X}_i)' \qquad (51)$$

The number of such discriminating functions is either equal to k(k-1), one for every pair of populations, or can be reduced to k-1 two-class problems, where the i-th problem is solved by a linear discriminant function that separates points assigned to π_i from those not assigned to π_i.[41]

Criterion for classification. Under the above assumptions, the classification rule is as the following:[42] Assign to group i, if for all j's

$$\chi_i^2 \geq \chi_j^2 + 2\ln(\,q_i/\,q_j), \text{ for } i,\ j=1,\ 2,\ \ldots,\ k \qquad (52)$$

where $\chi_i^2 = (X-\bar{X}_i)' \; S^{-1} \; (X-\bar{X}_i)$, and (53)

$$\chi_j^2 = (X-\bar{X}_j)' \; S^{-1} \; (X-\bar{X}_j) \qquad (54)$$

The rule may also be stated in the following form:[43]

Assign to group i, if

$$U_{ij}(X) > \ln\,(q_j / q_i), \text{ for } i,j = 1, 2, \ldots, k;\ i \neq j \qquad (55)$$

Since $U_{ij} = - U_{ij}$, there is no need to calculate more than k (k-1)/2 coefficient vectors of the functions. In fact, some of the coefficients can be calculated from others[44] and the number of independent functions is actually k-1.

Unequal dispersion matrices. The previous discussion embraced tests and classification rules pertaining to the k-group case under the assumption that the dispersion matrices of the groups are equal. If such an assumption does not correctly represent a real situation, the above tests and rules do not hold any longer. The appropriate treatment for the case of unequal dispersion matrices is as the following:

Test of significance. Anderson suggests a test statistic to test the hypothesis

$$H : \sum_{i=1}^{k} B_i \mu_i = \mu, \qquad (56)$$

where $B_1, B_2, \ldots, B_k$ are given scalars, μ_i is the mean vector of the i-th population, and μ is a given vector.[45] However, the desired hypothesis to be tested is

$$H_2 : \mu_1 = \mu_2 = \ldots = \mu_k \tag{57}$$

Eisenbeis and Avery show that H_2 hypothesis can be transformed to a form which is suitable for testing by the Anderson's test.[46] The proposed test is:

$$T^2 = N(\bar{Y}-\mu)' S^{-1} (\bar{Y}-\mu) \tag{58}$$

where S and $\bar{Y}$ are defined by

$$(N_1-1)S = \sum_{\alpha=1}^{N_1} (Y_\alpha-\bar{Y})(Y_\alpha-\bar{Y})' \tag{59}$$

$$Y_\alpha = B_1 X_{1\alpha} + \sum_{i=2}^{k} B_i \left(\frac{N_1}{N_i}\right)^{\frac{1}{2}} \left(X_{i\alpha} - \frac{1}{N_1}\sum_{\beta=1}^{N_1} X_{i\beta} + \frac{1}{\sqrt{N_i N_1}} \sum_{\gamma=1}^{N_i} X_{i\gamma}\right), \tag{60}$$

$$\bar{Y} = \frac{1}{N_1} \sum_{\alpha=1}^{N_1} Y_\alpha = \sum_{i=1}^{k} B_i X_i \tag{61}$$

$$\bar{X}_i = \frac{1}{N_i} \sum_{\beta=1}^{N_i} X_{i\beta} \tag{62}$$

$X_{i\alpha}$ is an observation vector from population i and N_i is its number of observation with N_1 being the smallest one. When H hypothesis is true the statistic has the T^2-distribution for dimension p, with N_1-1 degrees of freedom.

The function. When the dispersion matrices of the subject groups are not equal, the discriminant functions would be in quardratic forms. The procedure of constructing such functions, however, follows that of the linear case. And, utilization of the likelihood ratio yields k(k-1) functions of the following form:

$$U_{ij}(X) = \frac{1}{2}\ln\left|\frac{S_j}{S_i}\right| - \frac{1}{2}\left[(X-\bar{X}_i)' S_i^{-1}(X-\bar{X}_i) - (X-\bar{X}_j)' S_j^{-1}(X-\bar{X}_j)\right] \quad (63)$$

for i, j = 1, 2, ..., k; $i \neq j$

where S_i and S_j are the sample dispersion matrices of group g and h, respectively. An element of such matric is:

$$(S_i)_{gh} = \frac{1}{N_i - 1}\sum_{\alpha=1}^{N_1}(x_{ig\alpha}-\bar{X}_{ig})(X_{ih\alpha}-\bar{X}_{ih}), \text{ for } i=1, 2, \ldots, k \quad (64)$$

The difference between such function and its linear counterpart hinges upon the gradratic terms of $X' S_i^{-1} X$ and $X' S_j^{-1} X$ which result from the expansion of $(X-\bar{X}_i)' S_i^{-1} (X-\bar{X}_i)$ and $(X-\bar{X}_j)' S_j^{-1}(X-\bar{X}_j)$, respectively. In the case of equal dispersion, the quadratic terms are cancelled, leaving only the linear terms. This is not the case, however, when the dispersion matrices are not equal. The related functions, therefore, contain both linear and quadratic terms.

Criterion for classification. When the a priori probabilities of membership in populations are known the classification rule is as the following:[47]

Assign to group i, if

$$\chi_i^2 \leq \chi_j^2 - \ln\left|\frac{S_i}{S_j}\right| + 2\ln\frac{q_i}{q_j}, \text{ for } i, j = 1, \ldots, k \qquad (65)$$

where χ_i^2 and χ_j^2 are defined by

$$\chi_i^2 = (X-\overline{X}_i)' S_i^{-1} (X-\overline{X}_i) \qquad (66)$$

$$\chi_j^2 = (X-\overline{X}_j)' S_j^{-1} (X-\overline{X}_j) \qquad (67)$$

The rule may also be stated in terms of the functions:

Assign to group i, if

$$U_{ij}(X) > \ln (q_i / q_j) \qquad (68)$$

Relation to Regression Analysis

Researchers are usually more familiar with regression analysis than with multivariate discriminant analysis. In general, however, both approaches are characterized by several similar features. In his 1966 paper,[48] Ladd compared linear probability functions[49] and discriminant functions, and concluded that they "start from quite different places, follow different routes, and end up at nearly the same place."[50]

Fisher shows that by performing a regression analysis in which the dependent variable of the first group equals $N_2/(N_1+N_2)$ and that of the second group equals $-N_1/(N_1+N_2)$, the regression coefficients are proportional to those of the discriminant function.[51]

Such a regression analysis may be written in the following form, as set forth by Kendall:[52]

Source	Sum of Squares	Degrees of Freedom
Regression	$[N_1N_2/(N_1+N_2)]\,\Sigma\,\ell_i\,(\overline{X}_{1i}-\overline{X}_{2i})$	p
Residual	$[N_1N_2/(N_1+N_2)][1-\Sigma\ell_i(\overline{X}_{1i}-\overline{X}_{2i})]$	N_1+N_2-p-1
Total	$N_1N_2/(N_1+N_2)$	N_1+N_2-1

where l_i is the i-th coefficient of the regression line. It has been shown[53] that the R^2, the multiple correlation coefficient of the regression, calculated from the above table, has the following relation with D^2 of the corresponding discriminant function:

$$D^2 = \frac{R^2}{1-R^2} \cdot \frac{(N_1+N_2)(N_1+N_2-2)}{N_1N_2} \qquad (69)$$

The analysis of such regression yields the same F as does the D^2 analysis.

Cramer generalized the conclusion obtained by Fisher, as stated above, and concluded that the same relation holds for any two distinct values of the dependent variable.[54]

The Reliability Tests

As was discussed before, one of the purposes of constructing an MDA model is its application to assigning individual observations, drawn randomly from two or more populations, to their related groups. If the assignments are made correctly for most of the times, the model is said to be accurate. Otherwise, it does not embody sufficient discriminatory power to be of any value. There are several methods to test the reliability of the computed model. Four of these methods which are most often used are the following: (1) The Original Sample Method, (2) The Holdout Sample Method, (3) The Lachenbruch one-at-a-time Holdout Method, and (4) The Scrambled Sample Method.

The Original Sample Method. To determine the accuracy of the model, it has been a usual practice to set up a so-called confusion matrix, which was first introduced by C.A.B. Smith.[55] But, as noted by Frank, Massy, and Morrison,[56] this method underestimates the probability of misclassification. The bias is due to the fact that the observations which are used in constructing the classification matrix are the same observations which have been used to build up the discriminant model.

The Holdout Sample Method. An alternative method which has been devised to eliminate the above bias is the holdout method. In this method the sample is first divided into two parts. The first part is used to derive the discriminant function, and the second part is then classified by means of the function derived from the first part. The proportion of observations in the second part which have been misclassified constitutes the probability of misclassification. As noted by Lachenbruch and Mickey,[57] the main attraction of this method is the ease with which the error rates can be estimated along with the independence of the distribution of the estimates upon the form of the population. Its drawbacks, however, are its requirement for a large sample which may not be available in many cases, and its uneconomical use of data.

The Lachenbruch one-at-a-time Holdout Method. In an attempt to remove these deficiencies, Lachenbruch proposed another alternative.[58] His suggested method holds one observation at a time, computes a discriminant function with the remainder observations, classifies the omitted observation with the computed function, records the result, if the observation is misclassified, and repeats the procedure for all the individual observations. The number of misclassified observations obtained in this way is then used to compute the error estimates. Lachenbruch suggests that these estimates are unbiased for the probabilities of misclassification for a discriminant function.

The Scrambled Sample Method. When the available sample is too small to be split into two parts--one for the analysis and another for the validation, the scrambled sample method may be used for evaluating the validity of the discriminant model constructed from the entire available sample.

This method, which has been suggested by Frank, Massy, and Morrison,[59] employs the original sample and reassigns the individual observations to populations at random. Then the discriminant coefficients for this scrambled sample are computed, and the discriminating power of this model is estimated from its associated confusion matrix (classification table), as for the original sample. Since the individuals were assigned to the populations randomly, the discriminating power of this model should be zero. The deviation of its power from zero would be due to the sample bias[60] of the original sample.

To test the significance of the difference, binomial distribution theory may be used in the two-population case for the following t test:[61]

$$t = (Q-P)/\sigma_P, \tag{70}$$

where Q is the proportion of sample observations correctly classified by the discriminant analysis, P is the correct proportion one expects by chance, and $\sigma_P^2 = P(1-P)/N$.

Finally, the sample bias observed in this way can then be taken into account in evaluating the discriminatory power of the original model.[62]

The Relative Importance of Variables

In constructing a discriminant function the researcher wishes to include those variables which have the greatest discriminatory power and exclude those having the smallest. If there were no correlation between the variables,[63] ranking of the variables could be made by means of either the univariate F-test, which is based on the previously-discussed Wilks' lambda, or the weighted discriminant function coefficients.[64] The more general cases, however, are those in which the variables are not independent of each other. Unfortunately, for such cases, there is no test available yet which can determine the discriminatory power of a specific variable. It is, however, possible to evaluate the relative importance of the variable subsets.

The multivariate generalization of the univariate Wilks' lambda provides a means to rank all the variable subsets having the same size. For this purpose, an F-test is formed on the basis of the corresponding Wilks' lambda. It is then used in one or more of the following selecting procedures.

There are four stepwise procedures for evaluating the relative importance of various variable subsets. They are: (1) Conditional Deletion Selection, (2) Forward Selection, (3) Backward Selection, and (4) The Complete Selection.[65] But none of these procedures will necessarily yield the same ranking for each variable. The difference in the ranking is due to the correlation between variables coupled with the way in which variables are successively added to or deleted from the starting subsets in different procedures. With the exception of the last procedure, which searches among all combinations of a subset with a given size to find the "best" one, other procedures do not give a chance to a variable to re-enter into any subsequent subset, once it has been dropped from a current subset. Since inclusion of a variable with negative correlation is helpful in increasing the relative importance of a residual subset and exclusion of such variable deducts from its importance, and the opposite is true for the variables with positive correlations, changing the order of addition and deletion of the variables changes the ranking of a variable from one method to another.

At present, the preference of one procedure over another is largely a subjective judgment and depends on several factors. Some of these factors are the desired level of trade-off between accuracy and costs, the goal of the research, and the feasibility of employing a method under a set of constraints. Whereas the univariate F-test of individual variability is the crudest method, for example, it needs much less computer time than the complete selection method. While one procedure may yield a better result in maximizing the differences between group means, another procedure may give a better classification result. And, finally, the total number of variables may be a determining factor in employing the complete selection method for selecting the best subset with optimal size.

It should be noted that all the selection procedures are based on the assumptions that: (1) the data arise from multivariate normal populations, and (2) the associated dispersion matrices are equal. When these conditions are not met, the stepwise selection procedures are only approximate methods.

In the selection procedures discussed above, the importance of variable subsets is compared only for a given size, in each step. To compare subsets of different size, we need to have another procedure. Unfortunately, only a special kind of such procedure is available which has been given by Rao. In fact, his algorithm is a test of the hypothesis of sufficiency of subset of variables $(X_1, X_2, \ldots, X_q)$ or, equivalently, the hypothesis of the absence of additional information of the remaining subset $(X_{q+1}, \ldots, X_p)$. Following is his suggested F-statistic.[66]

$$F = \frac{N_1+N_2-p-1}{p-q} \cdot \frac{C(D_p^2-D_q^2)}{1+CD_q^2} \qquad (71)$$

where $C = N_1N_2/(N_1+N_2)(N_1+N_2-2)$, D_q^2 is the Mahalanobis D^2 statistic on the entire variables set, and D_q^2 is that on the selected subset.

Computer Programs

There are several MDA computer programs available either in a listing or in a package form. Each of these programs, however, has its own capability and limitations. Therefore, to make an efficient use of them, their characteristics should be known.

Cooley and Lohnes' Program. Perhaps one of the earliest MDA computer programs is that of Cooley and Lohnes which was published in 1962. It is a part of their book on multivariate procedures.[67] This program computes multiple-discriminate functions for up to 50 variables and any number of groups. Wilks' lambda test of the significance of discrimination and scaled vectors to show the relative contributions of the variables to each function are also computed. Their classification program, which is separate, computes discriminate scores and probability of group membership for up to 20 groups, 20 functions, and 50 variables.

BMDO4M.[68] This program computes a linear function for two groups with sample size of up to 300 in each group and a number of variables of up to 25. Its output includes mean scores, dispersion matrix and its inverse, function coefficients, D^2 statistic, z-values of the observations and their group mean and variance.

BMD05M.[69] This program computes a set of linear functions for up to five groups with sample size of up to

175 in each group and a number of variables of up to 25. It performs the group assignment, and its procedure is based only on the estimated probability density. It also evaluates the computed functions. The output of this program is similar to that of the previous one with additional information on classification and evaluation.

BMDO7M.[70] This program also computes a set of linear functions, but its main features are the stepwise procedure and plotting of the first two canonical variables. The number of variables and that of groups should not exceed 80. Its classification procedure uses prior probabilities. The output includes most of those mentioned in the previous program, with additional information for each step.

SAS DISCRIM Procedure.[71] This program performs discriminant analysis based either on the individual within group covariance matrices or on the pooled covariance matrix. The choice is either optional or according to the result of the homogeneity test. The procedure is direct, i.e., it does not use stepwise method. However, it uses the prior probabilities in classification. The output includes values of the classification variables, frequencies, and prior probabilities, simple descriptive statistics for each group, and some other optional information. It also prints classification results for each observation.

SPSS DISCRIMINANT Program.[72] This program performs discriminant analysis either directly or through stepwise methods. Its classification procedure may be optionally based on the separate group covariance matrices, rather than the pooled matrix. In addition to the descriptive statistics, it optionally computes a test for the equality of the covariance matrices. Additional options control several other features. These include handling of missing values, plotting and punching discriminant scores, printing the classification function, printing and punching the classification results, and punching of various matrices.

Eisenbeis and Avery's MULDIS Program.[73] This is perhaps the most comprehensive program on MDA, so far. The listing of the output items, of itself, requires 3-4 pages. The available options are also of many varieties. These include features such as linear and quadratic approaches, complete, forward, and backward stepwise selection procedures, and Lachenbruch holdout classification method. The contribution of the authors comes not only from their program, but also from their excellent book which has been devoted entirely to the subject matter.

Application of MDA in Financial Area

In contrast to the early applications of the MDA in the areas of anthropology and plant taxonomy which was around 1920 and 1936, respectively, and with the exception of the pioneering work of David Durand[74] in 1941 on evaluating the credit worthiness of loan applicants by utilizing MDA technique, its use in the financial area came as late as 1959, when James Walter attempted to classify firms into high or low price-earnings ratio groupings.[75]

The work of Myers and Forgy on development of numerical credit evaluation systems,[76] that of Paul Smith on measuring risk in installment credit,[77] and that of Keith Smith on classification of investment securities[78] appeared in the next few years. These works are briefly discussed in the thesis of Altman, who made an extensive and successful use of MDA in predicting corporate bankruptcy,[79] in 1967. Altman's study has been influential and rightly deserves special mention.

The utilization of MDA in the financial area has been intensified during the last few years. A brief review of published, and a few unpublished, studies follows:

Carleton and Lerner[80] sought to develop a scoring system for municipal bonds which could duplicate the ratings of Moody's. For this purpose, they first considered five-group ratings. Then, they experimented with a two-group rating. By using a six-variable profile they obtained linear discriminant functions for each case. But their predictive efforts produced mixed results. The analysis, nevertheless, provided insights into the structure of bond ratings.

In another study, Renwick[81] employed a linear discriminant analysis to verify that investment portfolio behavior can be characterized and classified using a combination of four interrelated variables. The function he finally computed contained only three of the four variables, and was successful in discriminating between high performance portfolios and low performance portfolios.

Haslem and Longbrake[82] utilized a stepwise discriminant analysis to find a set of variables which can explain differences in bank profitability. In the final function, they used eight variables among which the ones related to operating expenses were particularly important. In developing the model, they neither tested for the equality of dispersion matrices nor used a holdout sample method. However, they tested the relevance of the variable set for all sample banks by the multiple regression analysis.

By using a linear discriminant function Williams and Goodman[83] concluded that financial variables do tend to distinguish between the various industrial classifications. According to their report, a set of 11 selected variables differentiates between utilities and industrials. Five selected industrial classes, i.e., chemical, drugs, domestic oils, steel, and electronics, also appear to be quite distinct by the help of 14 variables.

By employing a stepwise linear discriminant procedure and by using a small sample size of 42 firms, Edmister[84] built a model which was highly accurate in predicting small business failure in the original sample. For the validation purpose, however, he used the simulated sample approach.

The capital adequacy of commercial banks has also been a subject of a discriminant analysis. The linear model developed by Dince and Fortson[85] for this purpose contained five variables, and its validity was tested by an independent sample of size 38, which resulted in only about 8 percent misclassification.

Deaken[86] first replicated Beaver's study[87] with a new sample based on a univariate analysis, and then utilized Beaver's 14 financial ratios in constructing a linear discriminant function. He concluded that MDA can be used to predict business failures as far as three years in advance with a fairly high accuracy.

Lusk[88] explained by an example that the linear discriminant analysis can be effectively utilized for screening divisional investment proposals based on a set of characteristics which can be found in the firm's two ex-post accepted and rejected investment populations.

McCall and Eisenbeis[89] distinguished between a group of commercial banks which have, and a group of commercial banks which does not have, affiliation with saving banks. Their analysis indicated that an affiliated bank tended to have a lower participation rate in both the time and savings deposit and residential real estate loan markets than an unaffiliated bank. The result of their quadratic discriminant function showed that the two groups are distinct and overlap very little.

Pinches and Mingo[90] attempted to develop and test a model for predicting industrial bond ratings. Their final model, consisting of four linear functions, incorporated six variables and correctly predicted approximately 60 percent of the ratings for a holdout sample and another sample of newly rated bonds.

The objective of the study conducted by Bates[91] was to estimate a discriminant function which is more effective in identifying successful loan application from urban black enterpreneurs. He estimated two quadratic discriminant functions - one for black and one for white businessmen. His results suggest that the discriminant function which is appropriate for judging white borrowers does not effectively classify black borrowers.

Gilbert[92] employed a quadratic discriminant function to distinguish between decisions of banks to open and not to open de novo branch offices in the relevant markets within a three-year period following federal regulatory action on their merger applications. His model classified about 69 percent of his holdout sample correctly.

In searching for factors which make the holder of a bank charge-card an active or inactive user, Awh and Weters[93] employed a linear discriminant function. They reported that their discriminant function containing seven variables correctly classified 78 percent of the cardholders. However, they do not specify that this percentage has come from the original sample or from a secondary or holdout sample. There is also no mention about the homogeneity test of dispersion matrices of the two groups.

Trieschmann and Pinches[94] conducted a study to develop a model that will be able to statistically discriminate between distressed and non-distressed property-liability insurance firms. Their model consisted of a linear discriminant function with six financial variables. It could correctly classify 94 percent of the original sample. The validity of the model was checked by the simulated sample approach.

In evaluating the social (versus private) value of regular investment advice, Logue, Radcliffe, and Tuttle[95] undertook an empirical investigation to determine whether analysts' recommendations are more closely associated with future factors than with past factors, and to identify factors which impact most significantly in a recommendation. The results obtained from numerous stepwise linear discriminant functions distinguishing among various investment recommendation categories suggested that explanatory power is greater using the past data than future data. The power is even greater when both selected past and future variables are included.

NOTES

1. R.O. Duda and P.E. Hart, Pattern Classification and Scene Analysis, New York: John Wiley and Sons, Inc., 1973, p. 2.

2. Mahalanobis suggested $(\mu_1-\mu_2)' \Sigma^{-1} (\mu_1-\mu_2)$ as a measure of the distance between two groups, where μ_i, $(i=1, 2)$, is the mean vector of p variables characterizing the i-th population and Σ^{-1} is the inverse of the common variance-covariance matrix of the two populations. See T.W. Anderson, An Introduction to Multivariate Statistical Analysis, New York: John Wiley and Sons, Inc., 1958, p. 56.

3. For a test of normality, see R.V. Hogg and A.T. Craig, Introduction to Mathematical Statistics, New York: MacMillan and Co., 1965, pp. 343-8.

4. E.S. Gilbert, "On Discrimination Using Qualitative Variables," The Journal of American Statistical Association, Vol. 63 (1968), pp. 1399-1412. For other research work on this subject, see H. Linhart, "Techniques for Discriminant Analysis with Discrete Variables," Metrika, Vol. 2 (1959), pp. 138-149; also M. Hills, "Discrimination and Allocation with Discrete Data," Applied Statistics, Vol. 16 (1967), pp. 237-250.

5. See, for example, H. Myers and E.W. Forgy, "Development of Numerical Credit Evaluation Systems," Journal of American Statistical Association, Vol. 58 (1963), pp. 797-806; also, W. Carleton and E. Lerner, "Statistical Credit Scoring of Municipal Bonds," Journal of Money, Credit, and Banking, Vol. 1 (1969), pp. 750-764.

6. For further discussion and other advantages of Fisher's LDF, see Gilbert, op. cit., p. 1401.

7. R.A. Fisher, "The Use of Multiple Measurements in Taxonomic Problems," Annals of Eugenics, Vol. 7 (1936), pp. 179-188.

8. See M.G. Kendall, A Course in Multivariate Analysis, New York: Hafner Publishing Company, 1957, p. 111.

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13. In his invited address at the meeting of the Institute of Mathematical Statistics in Berkeley, California, June 16, 1949. Referred to in J.L. Hodges, Discriminatory Analysis, I. Survey of Discriminatory Analysis, Randolph Field, Texas: USAF School of Aviation Medicine, 1950, Mimeographed, 115 pages.

14. See footnotes 19 and 20, below.

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16. Hodges, op. cit.

17. H.O. Posten, Bibliography on Classification, Discrimination, Generalized Distance and Related Topics, Yorktown Heights, New York: International Business Machines Corporation, Thomas J. Watson Research Center, 1962, Mimeographed, 20 pages.

18. T.W. Anderson, S. Das Gupta, and G.P.H. Styan, A Bibliography of Multivariate Statistical Analysis, Edinburgh: Oliver and Boyd, 1972.

19. S. Das Gupta, "Theories and Methods in Classification: A Review," in Cacoullos, op. cit., pp. 77-137.

20. T. Cacoullos and G.P.H. Styan, A Bibliography of Discriminant Analysis, in T. Cacoullos, ed., ibid, pp. 375-434.

21. G.T. Toussaint, "Bibliography on Estimation of Misclassification," IEEE Transactions on Information Theory, Vol. 20 (1974), pp. 472-479.

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29. For a summary of these tests, see P.J. Rulon and W.D. Brooks, *On Statistical Tests of Group Differences*, Cambridge, Mass.: Educational Research Corporation, 1961.

30. See, for example, Fisher, *op. cit.*

31. Anderson, *op. cit.*, p. 109.

32. G.W. Ladd, "Linear Probability Functions and Discriminant Functions," *Econometrica*, Vol. 34 (1966), pp. 873-885.

33. Actually, Anderson first derived a corresponding classification rule for populations with known parameters; then he argued that when these parameters are not known, their replacement with sample estimates should give good results. See Anderson, *op. cit.*, 1958, p. 137.

34. *Ibid.*

35. Anderson, *ibid.*, pp. 118-120.

36. The following formulas follow those of Robert A. Eisenbeis and Robert B. Avery, *Discriminant Analysis*

and Classification Procedures, Lexington, Mass.: D.C. Heath and Company, 1972, pp. 16-20 and 98.

37. See W.W. Cooley and P.R. Lohnes, Multivariate Data Analysis, New York: John Wiley & Sons, Inc., 1971, p. 244; also M.M. Tatsuoka, Multivariate Analysis: Techniques for Educational and Psychological Research, New York: John Wiley & Sons, Inc., 1971, p. 162.

38. C.R. Rao, Advanced Statistical Method in Biometric Research, New York: Hafner Press, second reprint, 1974, pp. 260-264.

39. See Tatsuoka, op. cit., pp. 164-5; also, Rao, op. cit., pp. 272-3.

40. See Anderson, op. cit., 1958, pp. 149-150.

41. Both of these approaches can lead to regions with undefined classification. For an approach to avoid this problem, see Duda and Hart, op. cit., pp. 132-133.

42. Cooley and Lohnes, op. cit., p. 137.

43. Anderson, op. cit., 1958, p. 147.

44. In a three-group case, for example, $B_{23} = B_{13} - B_{12} = \Sigma^{-1} (\mu_1 - \mu_3) - \Sigma^{-1} (\mu_1 - \mu_2) = \Sigma^{-1} (\mu_2 - \mu_3)$. Ibid., p. 150.

45. Anderson, ibid., pp. 121-122.

46. Eisenbeis and Avery, op. cit., pp. 26-27.

47. Cooley and Lohnes, op. cit., p. 137.

48. Ladd, op. cit.

49. The term "linear probability function" is used to denote a regression function in which the dependent variable has the value of zero or one (ibid., p. 873).

50. Ibid., p. 884.

51. Fisher, op. cit., pp. 184-5.

52. Kendall, op. cit., p. 161.

53. See Ladd, op. cit., p. 883.

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61. Ibid., p. 253.

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63. The effect of correlation has been excellently explained by W.G. Cochran. See Cochran, "On the Performance of the Linear Discriminant Function," Technometrics, Vol. 6 (1964), pp. 179-191.

64. For detailed discussion on these two methods, see R.A. Eisenbeis, G.G. Gilbert, and R.B. Avery," Investigating the Relative Importance of Individual Variables and Variable Subsets in Discriminant Analysis," Communications in Statistics, Vol. 2 (1973), pp. 206-7.

65. For detailed information, see A.S. Goldberger, Econometric Method, New York: John Wiley and Sons, Inc., 1964; see also Eisenbeis, et al., ibid.

66. C.R. Rao, "Inference on Discriminant Function Coefficients," in R.C. Bose, et al., eds., Essays in Probability and Statistics, Chapel Hill: The University of North Carolina Press, 1970, Chapter 30, pp. 587-602.

67. Cooley and Lohnes, op. cit.

68. See W.J. Dixon, ed., BMD, Biomedical Computer Programs, Berkeley, Calif.: University of California Press, 1973, pp. 211-20.

69. See Dixon, *ibid.*, pp. 221-231.

70. P. Sampson, in Dixon, *ibid.*, pp. 223-253.

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72. See W. Klecka, in N.H. Nie *et al.*, eds., *Statistical Package for the Social Sciences (SPSS)*, New York: McGraw-Hill Company, Inc., 2nd ed., 1975, pp. 434-67.

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87. Beaver, *op. cit.*

88. E.J. Lusk, "Discriminant Analysis as Applied to the Resource Allocation," *Accounting Review*, Vol. 47 (1972), pp. 567-75.

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93. R.Y. Awh and D. Weters, "A Discriminant Analysis of Economic, Demographic, and Attitudinal Characteristics of Bank Charge-card Holders: A Case Study," *Journal of Finance*, Vol. 29 (1974), pp. 973-980.

94. Trieschmann and Pinches, *op. cit.*

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CHAPTER IV

THE EMPIRICAL TEST OF THE HYPOTHESIS AND PRESENTATION OF RESULTS

Introduction

In Chapter I, it was stated that one of the main purposes of this study is to identify economic and non-economic factors which might serve to differentiate between financially distressed securities brokerage/dealer firms and those which appear to be economically viable. To ensure this purpose, an analytical framework was set forth in Chapter II where it was hypothesized that the probability of financial distress of a firm at time t is a function of the net cash-flow per dollar of assets at time t, the extent of the risk assumed by and imposed upon the firm, and the mismanagement of the firm (equation 1 in Chapter II). It was also concluded that, because the specific relations between these elements and the probability of the financial distress could not be known, the proposed general relation should be expanded to include all possible factors affecting the above-mentioned elements, and then the expanded function should be exposed to a statistical test, using the MDA technique.

Having discussed the role of the MDA technique in differentiating between different groups in Chapter III, it is now time to empirically test the suggested hypothesis. But, to fully test the hypothesis, there is a need to have data on all the related factors included in the expanded form of equation (1) in Chapter II. Unfortunately, however, as reported in Chapter I, about half of the required data, specially with respect to the revenue and expense items, could not be obtained due to their confidentiality. As a consequence, the cash-flow element in the hypothesized function is now substituted with a more general term, the profitability. And the test, therefore, is conducted on a limited basis using only the information which is available. As will be shown later in this chapter, even with this limited information, the results are encouraging.

The chapter starts, after the introductory section, with the description of the source of data. Then, the method used in selecting the sample is explained. And, after specifying the data set, the subjects related to Test I are presented. This test is an experiment directed to test the modified hypothesis with the information contained in the last financial reports of the failed firms, which were issued, on the average, one year prior to their bankruptcy dates. Later, at the end of the chapter, a search will be made to determine the possibility of the extension of the methodology employed in Test I to a more remote time, i.e., two years prior to bankruptcy. This experiment forms Test II. The results of each test are presented at the completion of their related analyses.

Source of Data

As was briefly mentioned in Chapter I, the main source of financial data for the study was the Financial Questionnaire, Form X-17A-5. A small number of firms, however, had their Balance Sheets on file, instead, or had reported their financial position on Form X-17A-11. Thus, wherever Form X-17A-5 was not available, an attempt was made to obtain similar items of information from these other sources.

In using these sources, however, running into some problems was inevitable. A part of the missing data problem emanated from the use of different sources of data to obtain the same financial item. This point will be discussed later. The second problem is related to the wide variation existing between the reporting dates of the same firm. The time period elapsed between the last two reports ranged from 5 to 21 months (and in one case 23 months). This situation was particularly serious for the failed firms. For these firms, the mean value of the period was 12 months, but only 42 percent of the firms had exactly 12 months' elapsed time. The nonfailed firms showed less variation. The mean value of the period in this group was also 12 months, but the percentage of firms having exactly 12 months' elapsed time period were 73. In one experiment, an attempt was made to "normalize" these time variations. The method used and the result obtained will be discussed later in the section on "Potential Biases."

Another point to be noted here relates to the selection of appropriate years for data from the nonfailed firms. Although it might have been desirable to have the same number of financial reports from the nonfailed group in each year as from the failed group, the exact matching was not easily possible without introducing some biases. The bulk of the last reports,[1] nevertheless, is in the years 1971 and 1972 in both groups (65 percent in the failed group; 70 percent in the nonfailed group). Furthermore, the inclusion of GNP or similar variables (among the variable set), which takes into account the extent of the business activities in each year, may remove the remaining effects of such differences.

In addition to the above forms, some other sources of data were also utilized. Schedule D of Form BD (SEC form) was used to obtain some information about the president of the firm. Among other things, this form has sections on the age, education, and business experience, in the past ten years, of the person involved. Data on GNP, Dow-Jones and Standard and Poor's stock price indexes, market interest rates, and market value of the sales of stocks and lands were obtained from Statistical Abstract of the United States. Finally, revenue sharing documents and County and City Data Books were used for obtaining information on the population and per capita income of the places where the firms had their main offices.

An important point to be noted in connection with the financial reports is the wide variation which exists among the failed firms with respect to the last report lag time (LRLT) which is the time elapsed between the last report and the filing date of the SIPC application in court for liquidating the subject firms. As shown in Appendices A and C, the LRLT ranges from 2 months to as long as 32 months. Its mean, however, is about 11.5 months.

Selection of Samples

Failed Firms

In selecting broker/dealer firms to be included in the sample, attention was first paid to the failed group. This group's population consisted of 109 firms which had been placed in liquidation by SIPC during the four-year period ending December 31, 1974.[2] One of the characteristics of the firms which had a bearing on their selection was their number of years in business. The distribution of this factor for the failed-firm population is shown in Table 2.

For the purpose of this study, which incorporates, among other things, the standard deviation of some of the data items, it was desirable to have as many financial reports (annual in this case) on each firm as possible, and

TABLE 2. THE FAILED-FIRM POPULATION BY YEARS IN BUSINESS

Years in Business	Total No.	Total Percent	Cumulative No.	Cumulative Percent
0-1	5	4.59	5	4.59
1-2	25	22.94	30	27.53
2-3	21	19.27	51	46.80
3-4	20	18.35	71	65.15
4-5	14	12.84	85	77.99
5-10	14	12.84	99	90.83
10 and over	10	9.17	109	100.00
	109	100.00		

Source: SIPC Third Annual Report 1973, p. 20 and SIPC Fourth Annual Report 1974, pp. 18 and 28.

never less than two. But, of 109 failed firms, 79 firms were more than two years old and only 58 firms were more than three years in business. In addition, reviewing the files of the firms revealed that some of the firms either had not filed their last reports, or had some missing reports among their last three ones. Furthermore, during the data-collecting period some of the files were not available.

Under these circumstances, it was decided that the last two reports would be the minimum number of reports to be collected on failed firms and the last three the maximum appropriate ones. Thus, the following criteria were set to select the failed firm sample:

1. Availability of files.
2. Availability of two years or more of financial data.
3. No missing reports between last two or three reports.[3]

By using these criteria, 60 firms were selected and these firms actually exhausted the entire failed group population.[4] In the next step, the 60 firms were divided randomly into two equal-size groups. One group was used in the original sample by which the discriminant function is calculated (for the names and some characteristics of the firms in this group, see Appendix A). The other group was used in the holdout sample by which the reliability of the constructed model is tested (for names and some characteristics of the firms in this group, see Appendix C).

Nonfailed Firms

Selection of the nonfailed firms was much simpler than that of the failed ones. These firms were randomly selected from a SEC directory which contained the names of all the broker/dealer firms. There were, however, some limitations which required the adoption of the following criteria:

1. Existence of the selected firm, i.e., it has not been previously liquidated.
2. Availability of files.
3. Availability of financial data for at least three consecutive years.

By using the above criteria, 60 firms were selected to comprise the nonfailed group. These firms were, then, divided randomly into two equal-size groups. One group was used in the original sample (the names of the firms in this group are listed in Appendix B); and the other was used in the holdout sample (the names of the firms in this group are listed in Appendix D).

Data Set

After sample selection and collection of the related available Forms and other sources of information were completed, a schedule was prepared to obtain a certain set of data items from the collected material. This schedule incorporated several kinds of information, including: information on certain characteristics of the firm's president;

information about the financial report under consideration; data on certain financial items; information on non-financial characteristics of the firm; information on some aspects of the city of the firm's main business; and data on some indexes of the business activity during the reporting period. The exact items of the data set are listed in Appendix E.

Although an effort was made to incorporate only those data items which were common to most of the firms, occurrence of missing data in many places was inevitable. There were several causes for this problem. The first cause was the inavailability of the related source. The lack of Schedule D of Form BD, for example, was the reason that information on the president's characteristics could not be obtained for some firms. Replacement of Balance Sheet or Form X-17A-11 for Form X-17A-5 was the second one. Type of the firm was the third one.

In this regard, if the firm was a corporation, its retained earnings could be easily singled out, whereas separation of this item for some of the partnership and most of the sole proprietorship type of organizations was not possible without making some assumptions.[5] The total number of corporations, partnerships, and sole proprietorships in the failed group were, respectively, 54, 3, and 3; the numbers of such organizations in the nonfailed group were, respectively, 45, 8, and 7.

The last cause of the missing data problem was the method used by firms for reporting their financial items. While some firms had reported their data in detail, others had reported only totals. As an example, most firms had reported both "failed to deliver within 30 days" and "failed to deliver over 30 days." There were several firms, however, which had reported only the "total failed to deliver."

The missing-data problem is not, of course, something specific to the present study. In fact, in most cases, wherever an empirical research of this nature exists, the missing-data problem goes with it hand-in-hand. This situation has attracted the attention of many writers,[6] and several methods have been suggested for solving the problem. The easiest methods are either eliminating the observations which have the missing data, or omitting the variable in which missing data are found. But, neither of these methods fits all situations. If the number of observations is limited, it is both uneconomical and impractical to discard the whole set of data for the one or two missing items. The requirement of having a sufficient number of degrees of freedom also precludes such a treatment. The omission of variables may also be at times infeasible. This may be due to the theoretical importance attached to the subject variable.

Some other methods which have been suggested are Buck's regresssion method, Dear's principal component method, and mean substitution method. The present study has used mainly the mean substitution method for the treatment of missing values.[7] In doing so, each variable in each group (failed or nonfailed) and in each sample (original or hold-out) was considered separately, and the mean value of the variable, based on all available values in that group of that sample, was used for the missing values in that group of that sample only. The choice of the method has been due to the limitation in the number of observations, the analytical value of the variables, and the result of an experiment conducted by Chan and Dunn[8] which shows the superiority of the mean substitution method (and principal component method) to other methods.

Test I - One Year in Advance of Bankruptcy

Introduction

In the section on "Source of Data," it was stated that the last report lag time (LRLT) of the failed firms ranges from 2 months to 32 months, its mean value being 11.5 months. With this in mind, if the attempts in constructing a discriminating model which utilizes the information contained in the last reports prove to be successful in correctly differentiating most of the failed firms from nonfailed firms, the model's predictive span may be construed as being about one year. Building up such a model constitutes the first experiment, Test I.

Variable Set

The first step in constructing the experiment is the identification of the potential factors which might differentiate between financially distressed securities brokerage/dealer firms and those which appear to be economically viable.

If the collected data could have been exactly matched with the analytical framework set forth in Chapter II, there would have been no problem and no further discussion would have seemed necessary in this regard. Unfortunately, however, the limitation encountered on the data side has required some substitutions and reconsiderations of the relevant factors. However, for the most part, the same line of reasonings, as explained in Chapter II, has been followed in determining the variables to be included in the model.

A list of the variables selected in this way is included as Appendix F. Although the list has not been ordered in this way, each variable has its place in one of the general classifications of profitability, risk, and mismanagement or in one of the specific classifications thereunder, such as internal or external factors, as explained in Chapter II. In particular, variables related to the attributes of a firm's president, accounts failed to deliver and failed to receive, and the relative significance of the property and fixture accounts were specifically designed to proxy the mismanagement element of the hypothesized function.

Variables relating to the version of Tchebycheff's inequality and generally most of the variables in the form of coefficient of variation have been introduced to take the firm's risk exposure into account. Those variables which are traditionally considered as leverage factors also fall under risk category. And, the remaining variables, including those related to the general business activities and the characteristics of the city where a firm's business is, as well as the traditional variables affecting the profitability of a firm, fall in the profitability category.

At the time of reporting the final variable subset selected by the MDA model, a brief discussion will be made of some of those variables which showed up as relatively more important than the others. At this point, however, some explanation about variables X-50, X-51, and X-73 through X-82 seems in order.

When the concept of Tchebycheff's inequality (see equation 30 in Chapter II) was presented in the context of a measure of risk, it was concluded that this type of equation can be used to estimate the probability that the value of a random variable, such as retained earnings, falls at or below a disaster level. This idea was utilized in creating variables X-50, X-51, and X-73 through X-82, but with some modification which will be explained below.

Rather than using Tchebycheff's inequality in the original form, the reciprocal of its square root was utilized. More exactly, the variables were created by using $(E-d)/\sigma$ rather than using $\sigma^2/(E-d)^2$. There were two reasons for such a treatment: (1) As is seen, in the original version the expression (E-d) is in the denominator. It happens sometimes that E becomes equal to d, resulting in an undefined ratio. This problem is removed by inverting the fraction; (2) In the original version the expression (E-d) has been squared. This kills the negative sign of the result for the cases where $E<d$. The distinction between cases in which the mean value of the variable deviates from above or from below the disaster level is, however, of much value to the purpose of this study. Avoidance of the squaring, of course, solves this problem. The resulting version $(E-d)/\sigma$, may also be thought of as the standardized deviation of mean from the disaster level.

Another point of possible interest, with respect to the variable set, is the existence of high correlations between some of the variables, in all tests (see, for example, the total correlation matrix of Test I in Appendix G).

The extent of the relations ranges from perfect positive to perfect negative correlation. In some cases, the perfect positive correlations exist between those variables which have been based upon one significant common element of the data item. For example, X-7, X-8, and X-9 which have cash item in common, are perfectly positively correlated with each other. In other cases, the perfect positive correlations are found among the variables such as X-49, X-55, X-56, X-59, and X-84, which are indicators of the level of the general business activities. Finally, two examples of perfect negative correlation are the correlations between X-5 and X-50, and X-6 and X-51 which equal to -1, in both cases.

The existence of such high correlations between some variables necessitates omission of one of the variables from each correlated pair. This treatment is necessary because it eliminates computational and operational difficulties. If such variables are not removed from the analysis, negative square roots result during the process of inverting the dispersion matrices. Even if a little less than perfect correlation makes the convergence on the proper inverse matrix possible, the resulting relations may not be stable. In the present study, in all tests, one of the variables in the pairs having correlation coefficient equal to or greater than .87 have been omitted. The choice has been based principally on considerations, such as fewer missing data on and easier interpretation of the retained variables. The variables omitted due to the correlation have been marked by asterisks in the variable list in Appendix F.

A final point with respect to the variable set is the variable structure of total observations. A table including the mean, standard deviation, standard error of mean, maximum, and minimum of each variable in each failed group (60 firms) and nonfailed group (60 firms) is included, separately, in Appendix H. The univariate F-tests of the significance of mean separation between the two groups have also been included in this appendix.

Discriminant Analysis

In providing the groundwork for the present analysis, the theoretical aspects and general procedure for the utilization of the MDA technique were discussed in Chapter III, rather in detail. Here, the concern revolves around the specific procedure and relevant points in applying the method to the case of the brokerage/dealer firms.

Methodology. After all the relevant variables were determined, the original sample was used for deriving the discriminating function. Since the number of variables was 84 and the number of observations in each of the failed and nonfailed groups in the original sample was 30, a one-step method was not possible in computing the function. The maximum number of the variables which could be included in the analysis, while being able to conduct all the related statistical tests, was 28 (the number of observations in each group, 30, minus the number of groups, 2). Therefore, the following procedure was used in finding the "optimal" subset of variables.

Run 1. By utilizing the Eisenbeis-Avery MDA computer program,[9] the first 50 variables (excluding X-1, the firm's identification number) were first used for determining the correlated variables.

Run II. Then, the last 50 variables were run for the same purpose as above. The overlap was intended to obtain more information on the correlations. The results of these two runs led to the omission of 22 variables from further consideration, leaving 61 variables for the analysis (see Appendix F).

As was discussed in Chapter III, among the various procedures which can be used for selecting the last subset of variables, the complete stepwise method is the only one which can guarantee that the selected variables are the best ones. Yet, its use is limited to the cases where there are only a few variables. As the number of variables increases the required computing time becomes a forbidding problem.[10]

An alternative method which seems reasonably accurate and feasible is the midway method between backward or forward stepwise procedures and the complete stepwise method. It is not as good as the latter, but it improves the result of the former. This alternative which has been used in the present work combines the forward and backward stepwise procedures in the last part of the analysis.

Run III. By using the forward stepwise procedure, the first 28 variables (disregarding the omitted variables) were run and the "best" 14 variables from this subset were selected.

Run IV. The same procedure as above was used for obtaining the second 14-variable subset. This left 5 variables unexamined at this point.

Run V. The 28 selected variables obtained from Runs III and IV were run by a forward stepwise procedure and the "best" 14 variables were selected.

Run VI. The same 28 variables were then run by a backward stepwise procedure and again the "best" 14 variables were selected.

The results of Run V and Run VI are tabulated in Table III. Comparison of these results is useful in further screening of variables. As is seen in the table, 10 variables have been selected for both forward and backward procedures, 8 variables by one or the other procedure, and the remaining 10 by neither of them. By excluding these last 10 variables from further analysis another step is taken toward the final selection. The logic behind this treatment is that if a variable does not appear in the selected 14 variables by either the forward selection method or the backward selection procedure, it signifies that the variable most likely lacks any discriminant power.

Run VII. This last run contained the 10 variables common to both procedures, the 8 variables selected by only one or the other procedure, and the 5 unexamined variables left from Run II. These 23 variables were run by using the

TABLE 3. RESULTS OF THE FORWARD AND BACKWARD SELECTIONS IN RUNS V AND VI

Forward Selection	Backward Selection	Common Variables	Last Run Variables	Last 12 Variables Selected
X-5			X-5	X-5
X-6	X-6	X-6	X-6	X-6
	X-10		X-10	
X-12	X-12	X-12	X-12	X-12
X-15	X-15	X-15	X-15	X-15
X-18	X-18	X-18	X-18	X-18
X-22			X-22	X-22
	X-41		X-41	
X-43	X-43	X-43	X-43	
X-45			X-45	X-45
	X-47		X-47	
X-52	X-52	X-52	X-52	X-52
X-58			X-58	X-58
X-65	X-65	X-65	X-65	
	X-68		X-68	
X-69	X-69	X-69	X-69	X-69
X-73	X-73	X-73	X-73	
X-78	X-78	X-73	X-78	X-78
			X-79	
			X-80	
			X-81	X-81
			X-82	
			X-83	

forward selection method, and a subset of 12 variables were finally selected.

The choice of 12 has been due to compromise between accuracy and complexity of the model. The criterion for accuracy was Rao's F test, and the measure for complexity was inclusion of additional variables. The percentage significance level of Rao's F test[11] serves as a relative indicator of the reduction in discriminatory power of the entire variables set by using each smaller subsets. This significance level for the 12 variable subset was 99.6 percent, whereas the related values for 11-variable subset and 13-variable subset were, respectively, 93.7 percent and 99.8 percent. Selection of 12 variables seems, therefore, to be a reasonable trade-off between accuracy and complexity when the gain in precision from 11 to 12 variables (5.9 percent) is compared with the loss in precision from 13 to 12 variables (0.2 percent).

The selected variables. Thus, as a result of the above procedure, 12 variables were selected to construct the discriminating model. These variables, along with some additional information about their properties, have been included in Table 4. But, before the results of such selection are analyzed, some explanation of the reasons for including these variables seems necessary.

As mentioned earlier, each of the variables included in the analysis has its place in one of the three categories corresponding to the three elements in the hypothesized function. Although some of the variables may appear to belong to more than one group, the intention has been to include them in a category where their differentiating power seems to operate more directly and forcefully.

Profitability. Variable X-12, which is the ratio of retained earnings to total assets clearly falls in this category. The higher the profit is, the more likely that the ratio becomes larger. Percentage change in equity capital, X-45, also signals the extent of profitability. If the perspective of the firm's business is promising the capital is usually increased. In retrospect, also, a fruitful year increases the proportion of the equity capital with respect to the previous one. The reverse is, of course, true for a gloomy perspective and a year full of losses. Furthermore, the sign of the ratio indicates the direction of the change. Positive profit is most likely accompanied by a positive percentage change in equity capital and negative profit by a negative change.

Other variables which belong to this category are X-5 and X-58. The relation of a firm's age (X-5) to its profitability and the role that the interest rate (X-58) plays in a firm's revenue and expense accounts have, however, been discussed in detail in Chapter II.

While most of the above variables are internal to the firm, that is, the firm has control of them, interest rate variable is external to the firm. The appearance of this external factor among the factors differentiating failed firms from nonfailed firms may be a further inducement for reconsideration of the traditional approach in solving a similar problem which utilizes the internal factors only.

Risk. Some of the variables which fall in this category are versions of the traditional financial ratios dealing with leverage. But, others are not that familiar. X-15 and X-18 need little explanation. It is, however, worthwhile to note the role which subordinated debt plays in these ratios. Subordinated debt is generally considered as a part of the firm's capital for indemnification of other creditors. Yet, it is viewed as a devastating liability by some of the equity owners. As reported in Chapter I, this very subordinated debt has put several firms in the red, due to untimely withdrawal or improper execution of its related agreement.

X-22 is the ratio of the sum of the partly secured, secured, and unsecured customers' securities accounts to the firm's total assets. Holding other things constant, a firm which conducts most of its securities business on the cash basis or deals mostly with fully secured accounts, in the case of margin transactions, bears less risk than another firm having relatively large volumes of partly secured or unsecured accounts.

X-52 and X-69 relate to the trading and investment accounts. Firms which rely more on commissions and other sources of revenue which are more stable are less subject to risk than those which concentrate more on trading of and investing in securities. The existence of volatility in some of these accounts, as measured by their coefficient of variation, probably adds to this risk. This may be due to such factors as the high riskiness, or gambling nature, of such accounts. Furthermore, the higher is the percentage of such accounts on short position, the higher becomes the risk that the firm is exposed to.

X-78 measures the risk associated with the deviation of the ratio of the partly-secured and unsecured accounts to total assets (X-22) from the related disaster level (being the mean of the same factor in the failed group firms). The higher is the deviation from the above, the higher becomes the risk exposure of the firm, and vice versa.

Mismanagement. The inclusion of X-6, the age of the firm's president, in the analysis, was based on the assumption that, holding other things constant, the ability of the president in managing the firm's affairs is directly related to his overall experience which in turn is a function of age. In reviewing the data on the failed and nonfailed groups, it was noticed that there were some differences between these groups in this regard. This point was also noted by the subcommittee on Commerce and Finance which recommends the following:[12]

> ...The subcommittee believes, however, that, in addition, uniform minimum age and experience qualifications should be established by the SEC and therefore recommends that sections 15 and 15A of the Securities Exchange Act be amended to direct the SEC to establish such minimum standards....

X-53, the ratio of the property and fixtures to total assets may also measure the ability of the management to put the available resources to their best use. If a relatively large part of such scarce resources is tied up in an asset which does not take its part in generating revenue, the ability of the management is questionable. Variable X-81 measuers the standardized deviation of the mean of X-53 from its related disaster level (calculated by averaging the same in the failed-firm group).

H_1 test of equality of dispersion matrices. Before the selected variables can be used for constructing a discriminant function, it is necessary to have knowledge about the equality of the dispersion matrices of the sampled group. As discussed in Chapter III, if these matrices are equal, the proper discriminant function is a linear one. Otherwise, a quadratic function should be used.

By using the Box's approach (see equations (1) through (10) of Chapter III), the computer program performed the H_1 test for determining such equality. With the first degree of freedom (greater mean square) equal to 78 and the second degree of freedom equal to 10622.8, the related F statistic equaled 6.068 which is significant at 0.0 percent level. Thus, the hypothesis that the matrices are equal is rejected, beyond any doubt. The proper discriminant function is, therefore, a quadratic one.

The discriminant function and its analysis. In Chapter III, it was stated that the MDA technique can be used in two capacities: (1) in analyzing the differences between two or more groups and making inferences, and (2) in assigning individuals to the group to which they have the greatest resemblance, according to a certain set of attributes, and making predictions.

For inferential use of the technique the linear discriminant function which is constructed by the 12 selected variables is utilized, despite the result of the H_1 test. The reason for such a choice will be explained shortly. For predictive use of the technique, as well as for validity tests, the quadratic function is used.

There are two reasons for using the linear, rather than the quadratic, discriminant function for the inferential purposes:

(1) The quadratic function with such a large number of variables is too complicated to make any meaningful analysis possible.

(2) As will be shown later, due to the existence of a large distance between the means of the two groups as well as the virtually nonexistent of overlap between the groups, there will be little difference between the results of the linear versus the quadratic procedure. In this regard, the trade-off between slightly less precision and disproportionately more complexity is well worthwhile.

It should also be noted that the inferential use of the linear function does not prevent the use of a quadratic function in classification; nor does it change any result thereunder.

As was stated in Chapter III, there are several alternative methods of computing the linear discriminant function. One way is to use the matrix of eigenvectors associated with the latent roots of the determinental equation $|Q-\lambda W| = 0$, where Q is the matrix of the weighted among-groups deviations sum of squares of X, and W is the matrix of pooled within-groups deviations sum of squares.[13] This is the method used by the computer program for presenting the inferential part of the model. For computing the classification equations in the test space,[14] however, a more direct method, similar to the one explained in Chapter III, has been used. But, so far as the functions computed in different ways are proportional to each other, the choice makes little difference. This is, of course, due to the fact that only the ratios of the coefficients of the function are important.

Table 4 shows the coefficients of the linear discriminant function against the related variables. The relative importance of each variable is also shown in an adjacent column. An analysis of these values in conjunction with the other values in the table may shed some light on the issues for which the present research was undertaken.

By now, it should have been established that one of the most significant features of the MDA is its ability to drastically reduce the dimensions of the problem on hand. In the present case, this reduction in dimensions has been performed through multiplying the coefficient of each variable by the value of that variable in each original observation, summing up the results for all the variables in the function, and arriving at a discriminant score for each observation. The centroid or the mean of such discriminant scores for the failed group amounted to 0.03546 and that for the nonfailed group to 0.14356.

Thus, the lower discriminant score is associated with the failed group and higher scores with the nonfailed group. Therefore, if the contribution of a variable in the function is toward the increment in the discriminant score, the higher values of the variable are associated with nonfailure, whereas the lower values are related to failure. More specifically:

X-5 is the firm's age. Its coefficient is positive. Thus, the higher the age is, the greater becomes the

TABLE 4. A PROFILE OF THE SELECTED VARIABLES OF TEST 1.

Variable	Discriminant Function Coefficients	Relative Power of Variables	Rank	Failed Group		Nonfailed Group		Univariate F ratio*
				Mean	Standard Deviation	Mean	Standard Deviation	
X-5	0.00125	6.43%	8	6.20	4.82	12.87	8.11	14.97
X-6	0.00118	9.19	6	39.21	8.01	53.44	11.91	29.52
X-12	0.04812	12.24	2	-0.16	0.42	0.23	0.21	21.20
X-15	0.01118	2.01	12	0.32	0.17	0.62	0.28	24.73
X-18	-0.01561	2.80	11	0.68	0.17	0.38	0.28	24.73
X-22	-0.36912	9.66	5	0.03	0.05	0.01	0.00	9.99
X-45	-0.01969	8.70	7	0.32	0.77	0.08	0.26	2.63
X-52	-0.86764	10.23	4	0.02	0.02	0.00	0.01	24.45
X-58	0.00759	6.29	9	7.19	1.08	7.61	1.07	2.35
X-69	-0.32837	15.78	1	0.07	0.07	0.03	0.59	5.81
X-78	-0.00015	12.19	3	-1.99	5.09	-44.89	146.29	2.58
X-81	-0.00014	4.51	10	-2.08	6.26	-13.93	60.59	1.14

Note: Most of the figures in the table have been rounded from 7 or more decimal places.

*$F_{1,28}$ (.01) = 7.64; $F_{1,28}$ (.05) = 4.20; $F_{1,28}$ (.10) = 2.89.

discriminant score, and the higher becomes the probability that the firm falls in the nonfailed group. The reverse is, of course, true. The lower the age, the smaller becomes the discriminant score, and the higher becomes the probability that the firm falls in the failed group. In making inferences, the logic is reversed: Given that the firm is a failed one, and holding other things constant, its low discriminant score is associated with its low age. This result is congruent with what was expected and consistent with what was found by the NASD study. The variable, however, accounts for only 6.43 percent of the total discriminatory power of the function and ranks 8 among the 12 variables.

The variable associated with the age of the firm's president, X-6, also shares the same attributes assigned to that of the firm's age. That is, the failed firms show that their presidents were, on average, younger than the nonfailed frims. This is confirmed by looking in the columns of the table which show the mean values of the variables in each group. As is seen there, the mean for the failed group is 39.21 years, whereas it is 53.44 years for the nonfailed group. The univariate F test ratio for the differences between these two means is 29.52 which is significant at the .001 level and ranks first among the other variables, in this regard.

X-12, the ratio of retained earnings to total assets, has also a positive relation to the discriminant score. It shows that the nonfailed firms enjoy a higher ratio, whereas the failed firms are plagued by a low ratio. In fact, the mean of this ratio in the failed group is negative (-0.16) which indicates that the failed firms have, on average, negative retained earnings or a deficit which resulted from cumulative losses. On the other hand, the mean of the ratio in the nonfailed group is as large as 0.23. This means that the retained earnings of the sample firms in the nonfailed group constitute about 23 percent of their total assets. This, in turn, indicates the profit-making ability of the nonfailed firms. This variable is highly significant both on the univariate basis and in combination with other variables. Its univariate F ratio is 21.20 which is significant at the .001 level. In the discriminant function it ranks second, and accounts for 12.24 percent of the total discriminatory power of the function. The importance of this variable in the present case is consistent with the finding of the NASD study in this regard.

The relation between X-15 and the discriminant score is also positive. But, despite its second place on the univariate basis, it ranks last in the function and contributes only about 2 percent of the total discriminatory power. This variable, nevertheless, shows that nonfailed firms possess broader bases of equity capital and subordinated debt than their failed counterparts.

X-18 shares similar attributes ascribed to X-15 with respect to its rank and contribution. Its coefficient is, however, negative, indicating that higher values of the variables are associated with failed firms and lower values with nonfailed firms. In fact, the related mean value of the failed group is almost twice that of the nonfailed group. Verbalizing, the failed firms rely much more on financial leverage (excluding subordinated debt) than the nonfailed firms. In reference to the exclusion of subordinated debt, it may be worthwhile to note its "noise-" reducing effect in the ratio. This effect is well understood if it is noted that variable X-19, which is the ratio of total liabilities (being total assets minus owners' equity) to total assets, has not been selected by the procedure.

As a leverage measure, the NASD study has found that the ratio of (Total Liabilities + Subordinated Loans)/Owners' Equity is a significant variable. Its equivalent variable in the present study, i.e., X-14, however, has not been selected by the procedure in this test. Nevertheless, the conclusion of the NASD study with respect to the higher leverage (nearly twice) of the failed firms is consistent with the above findings.

The ratio of the partly secured and unsecured accounts to total assets, X-22, is inversely related to the discriminant score, as indicated by the negative coefficient. Thus, the failed firms in the analysis have demonstrated that they have had proportionately more customer securities accounts which have been partly secured or not secured at all than the nonfailed firms. This variable is fairly significant in differentiating between the two groups, and ranks fifth among other variables.

The coefficient of X-45, the percentage change in equity capital, is also negative. The negative sign indicates that the change is more pronounced in the failed firms than nonfailed firms. In fact, the mean of the ratio in the failed group is four times larger than that in the nonfailed group. The variable accounts for 8.70 percent of the total discriminatory power of the function and ranks seventh among other variables. But, its univariate F ratio is not significant at the 0.10 level.

X-52, which is the percentage of trading and investment accounts with credit balances, also has a negative coefficient. It may be inferred, therefore, that the failed firms are in a higher "short" position than their nonfailed counterparts. The rank of this variable is relatively high (fourth) on both univariate and multivariate bases and it contributes 10.23 percent of the total discriminatory power.

The effect of X-58, the interest rate in the year prior to the reporting date, has been negative on the failed firms and positive on the nonfailed firms. The reason for this situation may be the net position of these firms with respect to their interest income and interest charges. The performance of this variable is consistent with the behavior of X-18. This is the only external variable which has shown a relatively significant discriminatory power in the present test.

X-69, the coefficient of variation of X-41, is the first ranking variable which contributes 15.78 percent of the discriminatory power of the function. This and the next two variables are of the type suggested in Chapter II to be included in the analysis for revealing the differences between firms with respect to the volatility of their related factors and risk. The coefficient of the variable here is negative, indicating that the higher volatility in trading and investment accounts is associated with failed firms. The nonfailed firms in the sample have shown more stability in their trading and investment activities. It may also be of interest to note that variable X-41, of which X-69 is the coefficient of variation, has not been selected by the procedure due to its lack of discriminatory power.

X-78 is a version of Tchebycheff's inequality for the ratio of partly secured and unsecured accounts to total assets (X-22). As previously discussed, this ratio can be thought of as the standardized deviation of X-22 from its related disaster level. This variable has been shown to possess a relatively high discriminatory power (12.19 percent), and its rank is third among other variables. Its negative coefficient indicates that the higher values of the variable are associated with the failed firms and the lower values with the nonfailed firms. In fact, the mean of the variable in the failed group is -1.99 which is much larger than -44.89, the mean of the variable in the nonfailed group. This difference signifies that the deviation of the variable from its related level is small in the failed firms (as it had been designed to be, by averaging the values of the variable in the failed group to obtain the disaster level), whereas the deviation in the nonfailed group is very large and from below.

It may be of interest to note that both X-22 and X-78 have appeared in the function (both with negative signs), but the discriminatory power of X-78 is greater than that of X-22. It may also be worthwhile to note that the variable's significance in the multivariate combination is in direct contrast to its insignificance on the univariate basis (its univariate F ratio is 2.58 which is not significant at the 0.10 level).

X-81 is of the same type as X-78. It can be thought of as a measure of the standardized deviation of X-53 from its related disaster level. X-53, which has not appeared in the function due to the lack of any discriminatory power, is the ratio of property and fixtures to total assets. X-81, by its negative coefficient, indicates the association of its larger values with failure. It shows that the percentage of total assets of a nonfailed firm tied up in property and fixtures is significantly below the level that a failed firm, on average maintains. This may signify the ability of nonfailed firms and inability of the failed firms in appropriate management of their limited available resources. This variable also is not significant on the univariate basis, but ranks tenth in the multivariate combination, and contributes 4.51 percent of the total discriminatory power of the function.

The differences between the group means. After the "best" feasible subset of variables was selected, the H_2 test described in Chapter III was conducted to see whether the differences between the means of these variables in the two groups is significant or not.

From among several methods available for this purpose, two were utilized. The first one was the Rao's test which follows equation (42) in Chapter III. The second one was Hotelling's T^2 which has been defined in equation (12) of the same chapter. The results of conducting these tests are reported below.

Rao's test. Rao's test of equality of group means is an F test which relies upon the Wilks lambda. The calculated F statistic of this test with 12 and 47 degrees of freedom was amounted to 20.96101 which is significant at the 0.64669 E-12 percent level. And, the Wilks lambda for the test was 0.1574367.

Hotelling's T^2 test. This test has also an F distribution and is suitable for the two-group case. The related F statistic, with 12 and 47 degrees of freedom, equaled 20.97496 which is significant at the 0.6385504 E-2 percent level.

The overlap between groups. In the previous paragraphs, it was concluded that the differences between group means are highly significant. Although the existence of a relatively large distance between the group means is necessary to obtain the best predictive performance of the technique, it does not always guarantee that the designed classification scheme is successful. The cause is, of course, a possible overlap between groups. To evaluate the extent of success, it is necessary to examine the extent of the overlap.

The Eisenbeis and Avery computer program computed the following overlap table, Table 5, by using an appropriate chi-square score.[15]

TABLE 5. THE OVERLAP BETWEEN GROUPS IN TEST 1.

Group	Failed	Nonfailed
Failed	100.00	0.00
Nonfailed	0.4939302E-23	100.00

This table shows that about zero percent, or more exactly 0.4939302E-23 percent of observations of the failed group can be expected to lie farther from the center of the failed group than the mean of the nonfailed group. Similarly, it is expected that zero percent of the observations of the nonfailed group lie farther from their own mean than the mean of the failed group. Thus, there is in fact no overlap between the groups in the test space, given unequal dispersion matrices.

Validity Tests

To see how valid are the inferences made by the above analysis as well as to find out how well the model is useful for prediction, two tests were conducted: one with the original sample, and another with the holdout sample. Both tests were also run once with a quadratic classification method and once with a linear classification procedure.

Although the proper procedure is the quadratic one, according to the result of the H_1 test, inclusion of the linear method seems to be appropriate for the following reasons:

(1) Comparing to the quadratic equation, the linear equation is very simple and helpful in understanding the procedure.

(2) Examination of both methods is useful in comparing the results which are obtainable from each method.

(3) For the present case, the linear method seems to be as valid as the quadratic one. This is, of course, due to the existence of large distances between group means and lack of any significant overlap between the groups.

The equations used to run the validity tests are seen in the next page. The linear equation is based on equation (18) of Chapter III and the quadratic one on the equation (35) of the same chapter. To construct these equations there was a need to make some assumptions with respect to the *a priori* probabilities of drawing an observation from the populations of failed and nonfailed firms, and the cost of misclassifying an observation from the failed population as from the nonfailed and that of misclassifying an observation from the nonfailed population as from the failed. The effect of these assumptions comes through the cut-off point which is adjustable at any time, and does not seem to be as crucial as other features of the technique.

Regarding the *a priori* probabilities, it is desirable to use the ratio of the total number of the individual firms in both populations as a proxy to the *a priori* probabilities. If this method is not practical, similar results can be obtained by calculating the same ratio from a random sampling procedure. Neither of these methods was possible, however, for the present study, because, as was mentioned previously,[16] no comprehensive listing is available which includes the names of those firms which have voluntarily liquidated or gone out of business with no customers' interest involved.

The assumption made here is the equality of the *a priori* probabilities ratio with the ratio of the size of each group in the classificatory sample. For the present case, where the group sizes are equal, this assumption is equivalent to assuming equal *a priori* probabilities. The reason for such choice is to preserve the mere chance ratio which actually exists if an observation is drawn randomly from the classificatory sample. By doing this, the effect of incorrect estimation of the *a priori* probability is minimized and the validity of other aspects of the model is better revealed.

With respect to the cost of misclassification in each group, there is no firm foundation, for the present case, upon which a judgment can be based. It cannot be determined with certainty from whose point of view the costs should be considered. Should it be considered from the government's point of view, or should the views of any other interested party, such as the firm, creditors, customers, employees, investors, other firms, or the analyst, be taken into account? Even if a decision can be made in this regard, the problem is still too complicated to be solved. From the viewpoint of the government, for example, is it more equitable to let any ailing firm continue its existence as long as possible and violate the principle of the allocation of scarce resources by incorrectly classifying the firm into the healthy group; or, it is more fair to disrupt a viable going concern and endanger the principle of the free market enterprises by making an erroneous decision in putting the firm in a failed group? Even if a decision can be made in this regard, how can the associated costs be determined? And, in any way, the issue is mostly a subjective matter in the present case.

Due to the above difficulties, the costs of the misclassification in both groups of firms are assumed to be equal here.

Classification of the original sample.[17] Table 6 following shows results of reclassifying the observations in the original sample by both quadratic and linear procedures. The classification is performed by calculating the discriminant score of each observation and comparing the result with the specified cut-off point. The scores are calculated by subtracting χ_2^2 from χ_1^2 which are obtained by applying an appropriate procedure (linear or quadratic). The result is then compared with 0.0 for the linear equation, and -0.88367 for the quadratic equation and classification is done according to the rules stated under the equations. The discriminant scores calculated, and the group assignment made, by the quadratic procedure have been included in Appendices A and B against each firm's name.

EQUATIONS USED FOR CLASSIFICATIONS IN TEST I

```
LINEAR AND QUADRATIC EQUATIONS IN TEST SPACE

  1.  LINEAR EQUATION U(FLD, NFLD) =

             VARIABLE X-5    VARIABLE X-6    VARIABLE X-12   VARIABLE X-15   VARIABLE X-18   VARIABLE X-22   VARIABLE X-45
             -0.23947        -0.22518         -9.2115        -0.32805          4.8457          70.661          3.7686

             VARIABLE X-52   VARIABLE X-58   VARIABLE X-69   VARIABLE X-78   VARIABLE X-81
               166.10         -1.4526          62.861          0.29287E-01     0.26021E-01

      CONSTANT 1 = 15.298

      LOG OF P = 0.0

  2.  QUADRATIC EQUATION U (FLD, NFLD) =

             LINEAR TERMS    VARIABLE X-5    VARIABLE X-6    VARIABLE X-12   VARIABLE X-15   VARIABLE X-18   VARIABLE X-22
      X-5    -1.6881          0.29267E-01
      X-6    -0.98394         0.44991E-02     0.13724E-01
      X-12  -18.603           0.52927         0.47605         -32.369
      X-15    0.48662E 07     3.3063         -0.63285          38.461        -0.24332E 07
      X-18    0.48663E 07     3.7207         -0.88441          21.789        -0.48666E 07    -0.24333E 07
      X-22     3018.9        80.090           5.5341           57.020          -10687.        -9790.4          -0.48706E 06
      X-45   42.771           0.40866        -0.29114          27.578        -75.549          -83.916          -659.66
      X-52   -934.28        -27.785          -3.5431         -165.17          3286.0          3892.3           0.11134E 06
      X-58   -30.123         -0.23955         0.16740        -0.57615E-01     30.339           35.946           838.10
      X-69   -211.71         -4.9881         -0.89518          -76.344         447.63          593.47          -1263.4
      X-78    0.94451        -0.18065E-01    -0.22220E-01     -0.71876         2.0730          1.1097          -3.1581
      X-81   -0.70065         0.16823E-01     0.20408E-01     1.2078           0.45297         0.67241E-02      5.8638
```

```
(SECTION 2)

             VARIABLE X-45   VARIABLE X-52   VARIABLE X-58   VARIABLE X-69   VARIABLE X-78   VARIABLE X-81

   X-45        -24.733
   X-52         543.42          -57278.
   X-58         4.1357          -277.19         -0.17988
   X-69         115.18          -2110.3          -45.416         -436.69
   X-78      -0.22507E-01       -0.36337        -0.18460          1.3013          0.65116E-01
   X-81      -0.14948            5.0863         -0.19329E-01     -1.1229         -0.36298E-01     0.51666E-01

   2 LOG OF P = 0.0

   CONSTANT 1 = -0.24330E 07

   LOG OF CONSTANT 2 = -0.88367

P = THE RATIO OF APRIORI PROBABILITIES OF GROUP J AND GROUP I AND/OR COSTS OF GROUP MEMBERSHIP

FOR QUADRATIC EQUATION CONSTANT 2 = DETERMINANT (DISPERSION MATRIX OF GROUP J)/DETERMINANT (DISPERSION MATRIX OF GROUP I)

QUADRATIC EQUATIONS U(I,J) ARE READ AS A HALF MATRIX OF COEFFICIENTS DEFINED BY THE PRODUCT OF THE ROW AND COLUMN
TERMS.  PLUS LINEAR TERMS STORED IN THE FIRST COLUMN.  PLUS CONSTANT 1.

QUADRATIC CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP I IF FOR ALL OTHER GROUPS J, THE EQUATION U(I,J) IS LESS THAN THE NATURAL LOG OF CONSTANT 2 MINUS 2
TIMES THE NATURAL LOG OF P

LINEAR EQUATIONS ARE READ AS A VECTOR OF COEFFICIENTS PLUS CONSTANT 1

LINEAR CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP I IF FOR ALL OTHER GROUPS J, THE EQUATION U(I,J) IS GREATER THAN THE NATURAL LOG OF P
```

TABLE 6. CLASSIFICATION OF THE ORIGINAL SAMPLE IN TEST 1

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	30 (100)	0 (0)	30 (100)	0 (0)
Nonfailed	30 (100)	1 (3.33)	29 (96.67)	0 (0)	30 (100)
Col. Total	60 (100)	31 (51.67)	29 (48.33)	30 (50)	30 (50)

Note: Figures in parentheses are percentages.

As seen in the table, the linear equation has been successful in reclassifying the original observations 100 percent correctly, whereas the quadratic equation has misclassified one observation from the nonfailed group into the failed group.[18] Thus, from the total number of 60 observations only one firm, or 1.67 percent, has been misclassified. The misclassification rate of the NASD study for the original sample, using quadratic method, was 9.9 percent. As seen in the table, the quadratic procedure showed a slight inclination toward the failed group. If it was large, it could be corrected by adjusting the cut-off point in such a way that the expected rate of misclassification becomes equal in both groups.

To provide a visual representation of the quadratic classification procedure, a one-dimensional graph has been prepared which maps the classified observations in accordance with their discriminant scores (see Chart I). The observations graphed are letter-coded according to group membership. Letter F designates observations from the failed group (or group 1), and letter N those from the nonfailed group (or group 2).

Classification of the holdout sample. As discussed in Chapter III, the results of the classification of the original sample are biased upward. An independent holdout sample provides a better means for evaluating the discriminatory or predictive power of the model. Just like

```
               F      F
*************************************************************
*-900     *-850     *-800     *-750     *-700     *-650



                         F
*************************************************************
*-600     *-550     *-500     *-450     *-400     *-350
                                                Cut-off
                                                Point
                                              F
        F                            F        F    F  F
      F F   F   F          F    F    F        F FF FFFF  FN
*************************************************************
*-300     *-250     *-200     *-150     *-100     *-50



           N    N
 NNN      NN    N
 NNNNN  N NNN NNNN      N                 N
*************************************************************
*  0      *  50     *100      *150      *200      *250
```

Note:

F indicates an observation from the failed group.
N indicates an observation from the nonfailed group.
Scale for the graph: one * =5.
6 Observations were off the graph to the left.
6 Observations were off the graph to the right.

CHART I. CLASSIFICATION OF THE ORIGINAL SAMPLE OF TEST I.

the original sample, the observations of this sample have been classified by using both linear and quadratic procedures. The individual discriminant scores of the quadratic procedure, along with its group assignments, have been included in Appendices C and D. The overall results of the classification are also shown in Table 7.

As is seen in the table, the quadratic equation has misclassified four firms from failed and four firms from nonfailed groups. The linear equation has misclassified one observation less. Although the results of the two procedures are comparable, the firms misclassified are not exactly the same. Both procedures have misclassified firms No. 21, 26, 27, and 29 of the failed group. But, the quadratic method has misclassified firms No. 42, 43, 55, and 58 from nonfailed group, whereas the linear procedure has misclassified firms No. 36, 42, and 43. As seen in Appendix C, two of the misclassified failed firms (Nos. 21 and 27) have a low age, a high last report lag time, and only two reports. Other firms, however, do not show obvious reasons.

The quadratic method has, on average, misclassified 13.3 percent of the holdout sample, and the linear method 11.6 percent. As discussed before, the similarity between the results of the two methods is due to the large distance between the means and virtually no overlap between the two groups.

TABLE 7. CLASSIFICATION OF THE HOLDOUT SAMPLE IN TEST 1

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	26 (86.67)	4 (13.33)	26 (86.67)	4 (13.33)
Nonfailed	30 (100)	4 (13.33)	26 (86.67)	3 (10)	27 (90)
Col. Total	60 (100)	30 (50)	30 (50)	29 (48.33)	31 (51.67)

Note: Figures in parentheses are percentages.

The results of the quadratic classification of the holdout sample have also been graphed in the same way as that for the original sample (see Chart 2).

Potential Biases

There are several possible sources of potential biases in the results obtained. These potential biases might have been worked in either upward or downward directions: ascribing a higher degree of accuracy to the model than what it actually possesses, or attributing a lower level of validity to the results than what they really deserve. The following paragraphs discuss where these biases might have come from, and wherever possible, test the significance of their effects on the results.

Potential biases due to the sample. As was reported earlier in this chapter, it has not been possible to include any non-SIPC failed firm in the study. This might have introduced some biases into the model. Furthermore, the limitations existed and the considerations made in the process of sample selection, as reported, might have brought in the effects of deviation from randomness.

To estimate the extent of such biases, a test was run by using the scrambled sample method.[19] Although this test is more suitable for samples of small sizes, where the possibility of using the holdout sample method, as performed above, is nil, its application to the present study seems

```
  F                            F          F               F
************************************************************
*-900     *-850     *-800     *-750     *-700     *-650


                                 F
************************************************************
*-600     *-550     *-500     *-450     *-400     *-350

                                                Cut-off
                                                Point
                                              F           N
    N   F F   F      F         F       F      F FFF NF F FFN
************************************************************
*-300     *-250     *-200     *-150     *-100     *-50


 N
 NN   NN
 NN NNFNNFFNN  N  NN     N         N            N
************************************************************
*  0      *50       *100      *150      *200      *250
```

Note:

F Indicates an observation from the failed group.
N Indicates an observation from the nonfailed group.
Scale for the graph: one * = 5 .
7 Observations were off the graph to the left.
7 Observations were off the graph to the right.

CHART II. CLASSIFICATION OF THE HOLDOUT SAMPLE OF TEST I.

appropriate, due to the existence of the conditions described above, and as an additional means of checking the validity of the analysis.

To run this test, all the 120 observations were scrambled and each of the individual observations was assigned to one of the two groups randomly. Then, following the same methodology used to get the original function, the discriminant function for this scrambled sample was obtained.[20] By using this function the entire group of observations was classified. The results of both linear and quadratic procedures were 47.5 percent misclassification.

Since the observations used in computing the function were assigned to the groups randomly, it was expected that the discriminatory power of the model would be zero. Equivalently, it was expected that, by pure chance, 50 percent of the observations are classified incorrectly.

To determine the significance of the difference between the actual result, 47.5 percent, and the expectation from pure chance, 50 percent, a t test was conducted by using equation (70) of Chapter III, with the following results.

$$t = (Q-P)/\sigma_P = (0.475 - 0.500)/0.0456431 = -0.55$$

The calculated value of t indicates that the difference between the results of the model and chance is not significant at any level lower than 0.6 (the tabulated value of t at the 0.6 level is 0.524). In other words, if there has been any sample bias in the model, it has not most probably been significant.

As a matter of possible interest, the result of a slightly different approach to the problem is also reported. This approach is based on the logic of the scrambled sample method, but in a reversed order. Rather than scrambling the observations first and deriving the function, and then applying the function to the observations in their actual groupings, the modified approach derives the function first from the observations in their actual groupings, and then applies the function to the scrambled observations. If the model does not have any sample biases it should not be able to classify the scrambled observations any better than by pure chance. The advantage of this modified approach to the original one is the elimination of the costs associated with deriving the function from the scrambled observations, because the other function would have to be computed anyway.

As was expected, the result of the modified approach was almost exactly the same as that of the original one. That is, the 12-variable function derived from the observations in their actual groupings misclassified 46.7 percent of the scrambled observations with the quadratic procedure and 50 percent with the linear one.[21] The t value for the quadratic method is -0.72 which is again not significant at any level lower than about 0.5.

Potential biases due to data. In the section on "Data Set," it was reported that the study had run into the difficulty of missing data due to the existence of several factors named therein. It was also stated that in solving this problem the justified method of mean substitution was utilized. Although the superiority of this method (and principal component method) has been shown experimentally,[22] the existence of some biases caused by this factor cannot be denied.

To estimate the extent of these biases it is desirable to derive a discriminating model which does not contain any missing values among its data set.

If all the firms having one or more missing values in their data sets are omitted, the total number of firms which remains from the original 60 failed firms would be 19. The remaining number from the original 60 nonfailed firms would be 39. If all the data items having one or more missing values in their related data sets are omitted, the total number of omitted variables would be 31. If, however, the two methods are compromised by omitting some of the data items in which the missing values most frequently occurred and then eliminating the firms with missing values in the remaining data items, a better result may be obtained.

Following this procedure, eight data items were eliminated first, resulting in the elimination of 11 variables.[23] In fact, the number of variables omitted in this regard would be six, because the other five variables would be eliminated due to the correlations, anyway. But, from these six variables,[24] two, X-22 and X-78, are ones which have been among the selected variables in Test 1.

Then, all the firms having one or more missing values in the remaining data items were omitted. This kept 22 failed firms and 10 nonfailed firms[25] out of the analysis.

With the remaining 38 failed firms, 50 nonfailed firms, and 55 variables, and using a similar methodology as the one used in Test 1,[26] a set of 12 variables was selected[27] and the related linear and quadratic equations were derived (see Appendix J).

As seen in Appendix I, half of the selected variables are the same variables as in Test 1. It may also be of interest to note that 7 of the 12 variables are either in the form of the coefficient of variation or the standardized deviation from a disaster level.

The signs of the discriminant coefficients of the six common variables and their economic analysis are consistent with those of Test 1. The coefficients of the other six variables are also congruent with what has been expected. That is, the average failed firm has shown lower level of retained earnings, higher level of cash and deposits, lower level of equity capital plus subordinated debt, higher level of failed to receive, larger variation in failed to deliver, and larger variation in total liabilities to equity capital ratio than those of its nonfailed counterpart.

The centroids of the failed and nonfailed groups were, respectively, 0.54869 and 1.04588. The result of the H_1 test of dispersion matrix equality was 5.83, which is significant at the 0.0 percent level. The H_2 test of equality of group means resulted in an F statistic equal to 10.20 which is significant at the 0.61846E-10 percent level. And the expected overlap of groups in the test space was almost zero, given unequal dispersion matrices.

The quadratic and linear classification procedures respectively reclassified the original observations with 11.36 percent and 9.09 percent rate of misclassification (see Appendix K). These compare to 1.67 percent and 0.0 percent in Test 1, and to 9.9 percent in the NASD study for the quadratic procedure. The Lachenbruch classification method,[28] however, classified the observations with 15.91 percent misclassification by the quadratic and 10.23 percent by the linear procedure (see Appendix L). The corresponding value of the quadratic procedure in the NASD study was 13.8 percent.

As was noted above, the results of the classification of the reduced sample which does not contain any missing data are not any better than those obtained in Test 1. In other words, if any bias has been introduced into the model by using the mean substitution method in treating the missing data, this bias has probably not been significant.

Potential biases due to the variation in reporting dates. In the section on "Source of Data," it was stated that the time period elapsed between two financial reports of the firms ranged from 5 to 21 months. It was thought that this variation might have some negative effects on the comparability of data both for the same firm during different years and for different firms during the same year.

An effort was made to remove this variation by "normalizing" the deviated cases. Accordingly, the following formula was used to assign new values to the data items of the same firm in its reports of the years prior to the last one, taking the last report as the base.

$$Y_{t-1} = 12\,\frac{(X_{t-1} - X_t)}{\text{ETP}} + X_t,$$

where t is the base year, Y_t is the new value of the data item in the year prior to the base year, X_t is the value of the item in the base year, X_{t-1} is the old value of the item in the year prior to the base year, and ETP is the elapsed time period between the two reporting dates.

The fraction on the right side of the above equation computes the average change of the value of the data item during one month. When this change is multiplied by 12, the total change during 12 months is obtained. By adding this total change to the base value, X_t, the new value of the item results. This procedure is repeated for all the data items in the report until the entire report of the previous year is "normalized." Then, the normalized report is taken as the base and the data items in the report prior to the normalized one are treated in the same way.

Using these normalized data, the entire process of Test I was repeated, and 12 variables were selected.[29] From among these variables 8 are the same as those selected in the original experiment and two of them, X-41 and X-68, are similar to X-69 and X-22, respectively, which were selected in the original model to deal with trading and investment accounts and partly secured and unsecured customers' securities accounts. The remaining two variables are X-28 and X-73, dealing with failed to deliver outstanding 30 days or more and cash accounts, respectively.

The coefficients of both X-28 and X-73 in the related discriminant function are negative. This signifies that the higher values of these variables are associated with the failed firms. In other words, the failed firms, on average, have shown a greater percentage rate (with respect to total assets) of securities failed to deliver 30 days or more than the nonfailed firms (this is not contrary to what is usually expected). They have also demonstrated a higher percentage of cash to total assets than their nonfailed counterparts. This sets one to thinking that the failed firms may not have utilized their available cash supplies appropriately and have kept them idle in excess of their day to day requirements.

When the validity of the "normalized" model was examined by the original sample method and the holdout sample method, it was found out that the model did not perform any better than the original one. In fact, the reverse was true. The results of these tests are shown in Appendices M and N.

As is seen in these appendices, the quadratic procedure[30] of the normalized model classified 96.7 percent of the original and 80 percent of the holdout sample correctly. These compare, respectively, with those of 98.3 percent and 86.7 percent correct classification in the original model. Although the result of the normalized model is also encouraging,[31] its performance, in terms of the rate of misclassification of the holdout sample, is not as good as that of the original model. This, of course, is not what was expected.

The lower discriminatory power of the normalized model may lead one to the conclusion that no bias exists due to the variation in reporting dates. This, however, may be a misinterpretation of the results. A possible factor responsible for the result is the procedure used for normalizing the data items.

The implied assumption in the above procedure is that the changes in the data item from the previous year to the next is monotone, and, therefore, the principle of proportionality holds. Although justified in this situation where there is no alternative, this assumption may be a little weak.

Potential biases due to the variable selection. This point was also mentioned during the description of the methodology. The large number of variables in relation to the small number of available observations necessitated that the selection process be mingled with some heuristic procedure. The result was, of course, the construction of a model which can not be guaranteed to be the best one, although optimal under the existing set of constraints. The possible bias in this respect is, therefore, on the negative direction. That is, if it has not been subjected to the restraints, especially with respect to the number of observations, the model might have produced a better result. The limitation in the number of reports on each firm may also be a source of this kind of bias. This problem prevents the means and standard deviations of several data items, on which many variables have been based, from becoming fully operative.

Results

The efforts directed toward building a MDA model for analyzing and predicting the failure of the securities brokerage/dealer firms from about a year in advance resulted in a discriminant function consisting of 12 variables which measure the profitability, risk exposure, and mismanagement of the firm. The model separates the two groups of failed and nonfailed firms in such a way that there is virtually no overlap between them and the distance between the group means is highly significant.

The model was very helpful in describing the attributes of the failed and nonfailed firms. The sample firms showed the association of lower ages of a firm and its president with failure. They also demonstrated that nonfailed firms, on average, have relatively greater retained earnings to total assets and larger equity capital plus subordinated debt to total assets. The interest rates in the prior year have also had a positive effect on the nonfailed firms. The relations between failure and percentage changes in equity capital, percent of the short positions in the trading and investment accounts, and the coefficient of variation of the ratio of the trading and investment accounts to total assets were shown to be positive. This direct relation also exists for variables dealing with partly secured and unsecured customers' securities accounts, total liabilities minus subordinated debt, and property and fixtures management.

Assuming that the classification of the selected variables into the three elements of the hypothesized function, i.e., profitability, risk, and mismanagement, is correct, the profitability variables account for 33.66 percent of the discriminatory power of the model.[32] The contributions of risk and mismanagement variables are, respectively, 52.67 percent and 13.70 percent. This indicates that in the short run risk element is the most significant determinant of the firm's survival. The importance of profitability is next and that of the mismanagement is last.

The accuracy and validity of the model were examined by several methods. The very low percentage (only 1.67 percent) of the misclassification of the original sample indicated that the model is significantly accurate. The exposure of the model to the holdout sample was used as a means of both the predictive ability and the validity of the model. Its discriminatory power was shown to be relatively high by correctly classifying about 86.7 percent of the holdout sample. Finally, the scrambled method, the Lachenbruch method, and the "normalized" model did not indicate that the potential biases in the model are significant.

Test II. Two Years in Advance of Bankruptcy

Introduction

The model presented in the previous experiment is the outcome of the effort made toward analyzing and predicting the financial position of a brokerage/dealer firm, based upon the information obtainable from the firm's last financial report which was issued about a year prior to bankruptcy. The natural extension of such endeavor is an attempt to build up another model capable of serving the same purpose for two years in advance of a potential bankruptcy. That is, the analysis and prediction should be based on information obtainable from the firm's next-to-the-last financial report.

Although having such a model is highly desirable, the expectation from the model is usually too much to be materialized for at least the next few years, when the economy may become more under control. The reason for inability to build a highly successful model is inherent in the nature of the securities business. Unlike most other industries, the securities industry is more subject to changes in the economy and the danger of failure is levered by the very high financial leverage (recently 1 to 12) which is a going practice in that industry. A firm which is in a sound position today may be defunct tomorrow! Every major downward change in the prices of the securities brings down with it several brokerage/dealer firms. As was mentioned earlier in this chapter, of 109 SIPC failed firms about 40 percent did not have a chance to survive long enough to produce even two financial reports.

With this caveat in mind, it is not expected that a model using the financial information of two years prior to bankruptcy would be highly successful. On the contrary, it seems reasonable to expect that every bit of information obtainable from such a less-than-successful model makes the effort worthwhile. This statement is particularly true for the present study which uses only about half of the pertinent financial information, because the other half was inaccessible.

Variable Set

The variable set used in this experiment is a subset of that used in Test I. The variables omitted from the original set are those incorporating the standard deviation of some of the data items. Specifically, variables excluded are X-50, X-51, and X-63 through X-82. The reason for such treatment is the availability of only two financial reports for 22 of the 60 failed firms,[33] which leaves only one financial report of such firms for the present test. For the same reason variables dealing with the changes of data items from one year to another were also eliminated. Finally, a number of additional exclusions of variables arose from the correlation between some of them. In all, from the original set of 83 variables, only 29 variables remained to be included in the present test.[34]

Discriminating Analysis

The analysis presented in this test is also based on the analytic framework discussed in Chapter II, and the MDA technique described in Chapter III.

Methodology. The methodology used here is the same as that employed in Test I. The only difference is the required adjustment made in the process to match the reduction in the number of variables.

The selected variables. The culmination of using a set of 29 variables and exposing the financial data of 30 failed and 30 nonfailed firms[35] of the original sample to the MDA technique was the selection of 12 variables whose names and properties appear in Table 8. From among these variables four are the same ones selected in Test I, and the rest have some similarity to those of the previous test.

To facilitate the final conclusion to be drawn from this analysis, it is useful to consider each of these variables in its place in one of the three categories corresponding to the three elements in the hypothesized function. Again, in some cases, there is no clear-cut line between these categories, and a variable may appear to belong to more than one category. But it is desirable to have the variable in the category where its discriminatory power seems to be overriding.

Profitability. Among the 12 variables there are 3 which fall in this category: X-5, X-11, and X-49. X-5, the age of the firm, has previously been discussed. The relation between X-11, the ratio of retained earnings to equity capital plus subordinated debt, and profitability is similar to X-12 which was discussed in Test I. And, X-49, the percentage change in GNP is an external factor which also affects the profitability.

Risk. The variables assigned to this category are X-15, X-17, X-33, X-41, and X-52. X-15, the ratio of equity capital plus subordinated debt to total assets, and X-52, the percentage of trading and investment accounts with credit balances, have already been discussed. X-17 is equivalent to the traditional debt-to-equity ratio and requires no discussion. X-41, which is the argument of X-69, has been covered during the discussion of the latter in Test I.

With respect to X-33, which is the ratio of customers' securities secured accounts to total assets, it seems reasonable to expect that a firm which has a relatively higher level of secured accounts bears less risk than the one having a lower level, other things being constant. This variable, however, may not effectively show the relative risk. A better measure may be the one which shows the percentage of the secured account in relation to the accounts receivable from customers rather than to total assets, which is the case with X-33. Creating such a variable, however, was not feasible, because some observations had zero accounts receivable from customers and this would make the ratio undefined.

Mismanagement. The remaining four variables, i.e., X-6, X-9, X-29, and X-30, fall in this category. X-6, the age of the firm's president, was discussed in Test I. X-9, the ratio of cash plus other deposits to total assets, is similar to X-7 which was covered in the discussion of X-73 in the Lachenbruch test. And X-29, the ratio of failed to deliver and receive over 30 days to total assets, is similar to X-28 which was discussed in the test with "normalized" data.

Regarding X-30, the ratio of the accounts receivable from customers to total assets, an argument can be made for its inclusion among the profitability factors since the easier the credit is, the more sales result, and, up to a point, the more profit is earned. But, in the securities business, where extraordinary credit concession does not seem to be necessary in stimulating sales, a relatively large ratio of accounts receivable from customers to total assets may signify the inability of the firm to collect its receivables in time.

$\underline{H_1 \text{ test of equality of dispersion matrices}}$. As before, this test was run to determine whether the linear or the quadratic procedure is the appropriate method to be used in constructing the discriminant function and the related classification rule. The result of the test, which is based on an F statistic with first degree of freedom equal to 78 and second degree of freedom equal to 10622.8, shows a value of 2.85 for F (significant at the 0.78597E-13 percent level) indicating that the dispersion matrices are not equal and the proper procedure is the quadratic one.

The discriminant function and its analysis. The 12 variables just explained were used by the computer program to build up the linear and quadratic functions. Following the same procedure used in Test I, the inferential use of the MDA technique for the present test is also made via the linear function. The quadratic procedure will be used, however, for the predictive purposes. The Linear classification results will also be reported for additional information and comparison.

Table 8 shows the coefficients of the linear discriminant function. The contents of this table match those of Table 4 in Test I. To infer the role and the relative importance of each variable in the function, the sign of its coefficient and the percentage of its discriminatory power is analyzed.

To begin with, it should be noted that the discriminant mean (or centroid) of the failed group in reduced space equals -0.02 and that of the nonfailed group equals 0.16. This indicates that firms having larger values for the variables with negative coefficients tend to make lower discriminant scores and fall in the failed group, whereas the reverse is true for the nonfailed group.

As seen in Table 8, the properties of X-5 and X-6 in this test are almost the same as those in the previous test. Therefore, there is no need for further discussion for these variables. With respect to X-9, its coefficient is negative indicating that the failed firms have relatively more idle cash and other deposits than their nonfailed counterparts. The difference between the means of this variable in the two groups, although small,

TABLE 8. A PROFILE OF THE SELECTED VARIABLES FOR TEST II

Variable	Discriminant Function Coefficient	Relative Power of Variables	Rank	Failed Group		Nonfailed Group		Univariate F Ratio*
				Mean	Standard Deviation	Mean	Standard Deviation	
X- 5	0.00340	7.79%	7	6.20	4.82	12.87	8.11	14.97
X- 6	0.00256	8.91	5	39.21	8.01	53.44	11.91	29.52
X- 9	-0.14633	12.18	1	0.31	0.23	0.25	0.25	0.90
X-11	0.04877	10.76	2	-0.22	0.80	0.38	0.43	12.88
X-15	0.08646	7.76	8	0.36	0.24	0.63	0.28	16.32
X-17	-0.01180	10.37	4	3.26	3.35	1.13	1.39	10.39
X-29	-0.13912	5.51	11	0.08	0.15	0.02	0.06	3.62
X-30	0.12439	8.14	6	0.14	0.21	0.11	0.17	0.26
X-33	-0.21383	10.43	3	0.09	0.19	0.02	0.05	3.45
X-41	-0.08346	6.91	10	0.22	0.19	0.28	0.28	1.00
X-49	-0.88027	7.34	9	0.04	0.02	0.03	0.02	1.46
X-52	-0.32586	3.90	12	0.02	0.05	0.06	0.02	3.43

Note: Most of the figures in the table have been rounded from 7 or more decimal places.

*$F_{1,28}(.01) = 7.64$; $F_{1,28}(.05) = 4.20$; $F_{1,28}(.10) = 2.89$.

corroborates this indication. This variable, by contributing about 12 percent of the discriminatory power of the function, ranks first among other variables. It is, however, interesting to note that its univariate F ratio is next to the lowest one and is not even significant at the 0.05 level.

X-11 is in the second place. Its coefficient is positive, indicating that the smaller ratios of retained earnings to equity capital and subordinated debt are associated with the failed firms. The smallness may come through one or a combination of three sources: smaller retained earnings, larger equity capital, and greater subordinated debt. The negative mean value of the variable in the failed group, however, removes this doubt and indicates that retained earnings is the overriding factor.

X-15 and X-52 show similar behaviors in this test as they did in the previous test. The difference is, however, in their relative importance both on multivariate and univariate basis.

The negative coefficient and other properties of X-17 which is equivalent to the traditional debt-to-equity ratio, is exactly consistent with what is usually expected from such a variable. It is both significant on the univariate basis and in combination with other variables. By contributing about 10 percent of the discriminatory power of the function, it ranks four among other variables.

X-29 also has a negative coefficient indicating that the failed firms in the sample have had, on average, relatively larger failed to deliver and failed to receive accounts outstanding 30 days or more. The inability of failed firms to manage their accounts in time is a factor which is usually expected.

X-30 and X-33 are the only two variables whose behavior in the present experiment can not be interpreted unambiguously. The possible reasons for such a situation may be the existence of a relatively high correlation (0.72) between them, their relatively large standard deviations, and the flaw in X-33 with respect to the inappropriateness of its denominator as discussed in the risk classification of the variables, above.

The behavior of X-41 is similar to X-52 as discussed in the previous test. With its negative coefficient it associates failure with more reliance on the trading and investment activities. This association is consistant with common sense, if the variable is considered as a measure of risk. But the mean values of the variable in the two groups do not corroborate this indication.

Finally, X-49, shows that failure has an inverse relation with percentage changes in GNP. The greater the change in GNP, the larger becomes the probability of failure. As shown in the table, the failed firms in the sample have, on average, seen relatively higher changes in

GNP during their next to last year of operation than their nonfailed counterparts.

The differences between the group means. The same two statistical tests, Rao's and Hotelling's, described in the previous experiment, were utilized in the present experiment. Both tests yielded a value of 17.31 for the F statistic with 12 and 47 degrees of freedom, which is significant at the 0.23017E-10 percent level. The results of these tests indicate that the failed and nonfailed groups, as differentiated by the attributes corresponding to the 12 selected variables, are separate from each other, and the distance between their means is significantly large.

The overlap between groups. As discussed in Test I, the following table can be used to show the extent of the group overlap.

TABLE 9. THE OVERLAP BETWEEN GROUPS IN TEST II

Group	Failed	Nonfailed
Failed	100.00	7.16542
Nonfailed	0.3946710E-2	100.00

Given unequal dispersion matrices, Table 9 shows that about zero percent of the observations of the failed group can be expected to lie farther from the failed group than the mean of the nonfailed group. Similarly, it is expected that about 7 percent of the observations of the nonfailed group lie farther from their own mean than the mean of the failed group. Thus, there is some overlap between the two groups in the test space, but it is not large.

Validity Tests

In Test I, a relatively detailed discussion was made of the reasons for and methods of conducting validity tests. This discussion, as well as the assumptions made in regard to the *a priori* probabilities and misclassification costs, holds true for the present experiment. Except where otherwise specified, the same procedure employed in the previous test is followed here.

Classification of the original sample. By using the computed linear and quadratic equations, along with their related classification rules (see next page), the observations in the original sample were reclassified. The results of this classification, which indicate the accuracy of the model, are presented in Table 10, below. This table shows that the quadratic equation, which is more relevant in the present case, has misclassified only two observations from the failed group and none from the nonfailed group. This is equivalent to an average rate of misclassification of about 3.33 percent. This rate for the linear equation is 1.67 percent.

Classification of the holdout sample. Application of the linear and quadratic equations to the observations of the holdout sample in this test did not turn out results

EQUATIONS USED FOR CLASSIFICATION IN TEST II

```
LINEAR AND QUADRATIC EQUATIONS IN TEST SPACE

 1.  LINEAR EQUATION U(FLD, NFLD) =

          VARIABLE X-5    VARIABLE X-6    VARIABLE X-9    VARIABLE X-11   VARIABLE X-15   VARIABLE X-17   VARIABLE X-29
          -0.33843        -0.25433         15.548         -4.8483         -8.5959          1.1727          13.831

          VARIABLE X-30   VARIABLE X-33   VARIABLE X-41   VARIABLE X-52   VARIABLE X-49
           -12.367          21.259          8.2973          32.397          87.518

          CONSTANT 1 = 7.0491

          LOG OF P = 0.0

 2.  QUADRATIC EQUATION U(FLD, NFLD) =

          LINEAR TERMS    VARIABLE X-5    VARIABLE X-6    VARIABLE X-9    VARIABLE X-11   VARIABLE X-15   VARIABLE X-17
    X-5     3.9405          0.58244E-01
    X-6    0.11802          0.18144E-01    0.13666E-01
    X-9    -149.80         -1.9399         -1.1617          57.880
    X-11    22.911          0.55451        0.27448          1.0504         -8.5529
    X-15    157.20         -4.9341         0.33460E-01      78.610         -27.080         -75.071
    X-17    23.669        -0.62835        -0.11987E-01     16.511          -5.2408         -33.977         -3.8914
    X-29   -142.74          1.9329         0.22187          130.80         -58.426         -12.598          1.2036
    X-30    97.497          2.2121         0.98080         -73.125          68.403         -140.32         -11.493
    X-33   -97.485         -4.5184         -1.1800          122.01         -67.254          119.79          18.106
    X-41   -74.445          0.86445       -0.18767          56.429          0.88471        -25.030          0.50330
    X-52    440.06         -20.551         -3.2158          426.25         -0.35660        -524.67         -66.906
    X-49   -1204.1         -6.1724         -9.3333          1015.3         -84.723          674.47          151.15
```

```
(SECTION 2)

            VARIABLE X-29   VARIABLE X-30   VARIABLE X-33   VARIABLE X-41   VARIABLE X-52   VARIABLE X-49
    X-29     -392.76
    X-30      180.01          251.01
    X-33     -269.55         -499.08         -122.41
    X-41      80.998          95.406         -87.849          48.598
    X-52      335.64         -705.60          1285.3          177.79         -4755.3
    X-49      891.22         -1000.7          1004.9          432.37          2921.0          3510.3

    2 LOG OF P = 0.0

    CONSTANT 1 = -29.529

    LOG OF CONSTANT 2 = -5.4109

P = THE RATIO  OF  APRIORI PROBABILITIES OF GROUP J AND GROUP I AND/OR COSTS OF GROUP MEMBERSHIP

FOR QUADRATIC EQUATIONS CONSTANT 2 = DETERMINANT (DISPERSION MATRIX OF GROUP J)/DETERMINANT (DISPERSION MATRIX OF
GROUP I)

QUADRATIC EQUATIONS U(I,J) ARE READ AS A HALF MATRIX OF COEFFICIENTS DEFINED BY THE PRODUCT OF THE ROW AND COLUMN
TERMS, PLUS LINEAR TERMS STORED IN THE FIRST COLUMN, PLUS CONSTANT 1.

QUADRATIC CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP I IF FOR ALL  OTHER  GROUPS J, THE EQUATION U(I,J) IS LESS THAN THE NATURAL LOG OF CONSTANT 2 MINUS
2 TIMES THE NATURAL LOG OF P

LINEAR EQUATIONS ARE READ AS A VECTOR OF COEFFICIENTS PLUS CONSTANT 1.

LINEAR CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP I IF FOR ALL OTHER GROUP J, THE EQUATION U(I,J) IS GREATER THAN THE NATURAL LOG OF P
```

TABLE 10 CLASSIFICATION OF THE ORIGINAL SAMPLE IN TEST II

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	28 (93.33)	2 (6.67)	29 (96.67)	1 (3.33)
Nonfailed	30 (100)	0 (0)	30 (100)	0 (0)	30 (100)
Col. Total	60 (100)	28 (48.67)	32 (53.33)	29 (48.33)	31 (51.67)

Note: Figures in parentheses are percentages.

as good as those in Test I. This, of course, was not unexpected, as discussed in the introductory section. As shown in Table 11, the quadratic equation has misclassified 43.3 percent of the failed group and 23.3 percent of the nonfailed group. On average, the expected rate of misclassification, as calculated from this sample, is 33.3 percent. Although this rate is not low, it is also not too high to render the model useless.

To determine the relative reliability of the model and to see how much its results differ from those obtainable by pure chance, the t test specified in equation (70) of Chapter III was applied to the misclassification rate.

If the discriminatory power of the model was equal to zero the misclassification rate should not have differed from 0.50.

$$t = (Q - P)/\sigma_P = (0.6667 - 0.5000)/0.0645492 = 2.583$$

The tabulated t's for 0.01 and 0.005 significant levels are, respectively, 2.326 and 2.576. The comparison of the computed t with the tabulated t's indicates that the results obtained by the model significantly differ from what is expected by pure chance. This, in turn, means that the model has some discriminatory power and provides some useful information.

Results

In an attempt to examine the possibility of the entension of the inferential and predictive use of the MDA

TABLE 11. CLASSIFICATION OF THE HOLDOUT SAMPLE IN TEST II

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	17 (56.67)	13 (43.33)	23 (76.67)	7 (23.33)
Nonfailed	30 (100)	7 (23.33)	23 (76.67)	11 (36.67)	19 (63.33)
Col. Total	60 (100)	24 (40)	36 (60)	34 (56.67)	26 (43.33)

Note: Figures in parentheses are percentages.

technique to two years prior to the bankruptcy of securities brokerage/dealer firms, a discriminant model was constructed which uses 12 variables. These variables measure the profitability, risk exposure, and mismanagement of the firms. Based on the attributes specified by the variables, the model separates the failed from nonfailed group with a small overlap but a significant distance between their means.

Except in a couple of cases, the model was useful in analyzing the financial positions of the firms and describing the attributes of the failed versus nonfailed firms. As in the previous test, the sample firms demonstrated inverse relationships between failure and the ages of a firm and its president. This performance is consistent with the expectation that the effects of the age factors should have continued from the previous year. It was also shown that the failed firms in the sample had lower ratios of retained earnings to equity capital plus subordinated debts, and equity capital plus subordinated debt to total assets. The model indicated the association of failure with relatively high levels of cash plus other deposits, trading and investment accounts, and failed to deliver and receive outstanding 30 days or more. The ratio of total liabilities minus subordinated debt to equity capital plus subordinated debt was also greater in the failed firms than in their nonfailed counterparts. Finally, the failed firms in the sample were adversely affected by the percentage changes in GNP.

The behaviors of the sample firms in terms of the properties specified by the above variables were consistent with the hypothesized performances. There were, however, some ambiguities in interpreting the coefficients of X-30 and X-33 in the discriminant function. This difficulty is, however, understandable if the statement in the introductory section is considered valid.

With respect to the accuracy and validity of the model, the 96.67 percent correct classification of the original sample observations and 33.33 percent misclassification of the holdout sample observations provide some criteria for evaluating the model's usefulness. The t-test conducted for examining the misclassification rate of the holdout sample, however, revealed that the performance of the model significantly differs from what can be expected from pure chance. And, therefore, the model possesses some useful discriminatory and predictive power.

NOTES

1. The last report criterion for such classification is due to the differences in the number of reports among the firms. Some firms have two reports; others have three.

2. For a list of names of these 109 firms, see SIPC, Third Annual Report 1973, p. 48, and SIPC, Fourth Annual Report 1974, pp. 18-31.

3. The only exception was Hill, Curtin and Ackroyd, Inc., for which only financial reports of 1970 and 1972 were available. A midreport was generated by using the proportionality relations and applying the procedure explained on page 219.

4. To be more exact, the subject population is the one which has been handled by SIPC during its four years of existence. But there might have been some other firms which have voluntarily liquidated or gone out of business with no customers' interest involved. Apparently neither SIPC, nor any of the regulatory or self-regulatory organizations is interested in such firms and thus, to the author's knowledge, there is no comprehensive listing available to show the names of such firms.

5. To solve the problem, it was decided that the difference between the beginning equity capital and the ending equity capital of the sole preprietorship and partnership could be used as a proxy for their retained earnings.

6. For an excellent review of the literature up to 1966, see A. A. Afifi and R. M. Elashoff, "Missing Observations in Multivariate Statistics I. Review the Literature," Journal of the American Statistical Association, Vol. 61 (1966), pp. 595-604. For more recent publication on the subject, see the references in the article referred to in note 8, below.

7. In some part of the study, i.e., the part related to the Lachenbruch classification method, a combination of the observation elimination method and variable elinimation method has also been used.

8. L. S. Chan and O. J. Dunn, "The Treatment of Missing Values in Discriminant Analysis-I. The Sampling Experiment," Journal of American Statistical Association, Vol. 67 (1972), pp. 473-477.

9. See the section on "Computer Programs" in Chapter III.

10. According to Eisenbeis-Avery, the program requires about 6 minutes of CPU time on an IBM 360-50 to select the optimal subset for each level of a 15-variable set, and time approximately doubles with the addition of another variable. (See R. A. Eisenbeis and R. B. Avery, op. cit., pp. 77, fn.).

11. See R. A. Eisenbeis and R. B. Avery, ibid., pp. 76-77.

12. U. S. House of Representatives, Securities Industry Study, Report of the Subcommittee on Commerce and Finance (Subcommittee Print), Washington, D. C.: U.S. Government Printing Office, August 23, 1972, pp. 22-23.

13. See R. A. Eisenbeis and R. B. Avery, op. cit., pp. 9, 63, and 92.

14. For information on the differences between test space and reduced space, see ibid., pp. 19-20, 56-63, and 98-99.

15. For more information, see ibid., pp. 11, 98, and 99.

16. See note 3, above.

17. For the theoretical discussion and biases of this method, see the section on "The Reliability Tests," in Chapter III.

18. The misclassified firm is firm no. 26 of the nonfailed group in the original sample. It has also been specified in the related column (Group Assignment) in Appendix B.

19. See the subsection on "The Scrambled Sample Method" in Chapter III.

20. The variables constructing the function are: X-10, X-21, X-27, X-28, X-33, X-41, X-44, X-63, X-65, X-80, X-82, and X-83. For analysis purposes, this set of variables is, of course, meaningless, because it has been derived from the scrambled observations.

21. The names of the 12 variables are: X-6, X-12, X-22, X-29, X-33, X-37, X-45, X-67, X-70, X-76, X-78, and X-81.

22. See note 8, above.

23. The 8 data items were: failed to deliver within 30 days; failed to deliver over 30 days; customers' cash accounts, secured accounts, partly secured accounts, and unsecured accounts; failed to receive within 30 days; and failed to receive over 30 days (see Appendix E). The 11 variables were: X-22, X-26, X-28, X-29, X-31, X-32, X-33, X-34, X-39, X-68, and X-78.

24. The six variables are: X-22, X-28, X-29, X-33, X-68, and X-78.

25. The omitted firms have been marked by asterisks in Appendicies A, B, C, and D.

26. Variable X-49 was used in both first and second runs of this experiment.

27. For the names and characteristics of the selected variables, see Appendix I.

28. For a discussion of the Lachenbruch method, see "The Reliability Tests" in Chapter III.

29. The variables selected are X-5, X-6, X-12, X-15, X-18, X-22, X-28, X-41, X-52, X-58, X-68, and X-73.

30. The H_1 Test of the "normalized" model resulted in an F statistic equal to 6.163 which is significant at the 0.0 percent level. This signifies that the appropriate procedure is the quadratic one. Nevertheless, for the reasons mentioned previously and for preserving the comparability of the models, the results of both linear and quadratic procedures are reported in Appendicies M and N.

31. Its H_2 test resulted an F statistic equal to 15.5 which is significant at the 0.16860E-9 percent level.

32. The relative discriminatory powers of profitability, risk, and mismanagement elements have been calculated by summing up the relative power of the related variables obtained from Table 4.

33. The firms having only two financial reports can be identified by referring to the related columns in Appendices A and C.

34. The 29 variables included in Test II are: X-2, X-3, X-4, X-5, X-6, X-9, X-10, X-11, X-14, X-15, X-17, X-20, X-21, X-22, X-23, X-29, X-30, X-31, X-33, X-37, X-38, X-40, X-41, X-49, X-52, X-53, X-60, X-61, and X-83.

35. The firms in the original and holdout samples in Test II are the same firms as in Test I. This treatment may be useful in extending the analysis somewhat further.

CHAPTER V

SUMMARY AND CONCLUSION

Summary

The important rule that the securities brokerage/dealer firms play in flowing funds in the nation's capital market by channeling the savings of individuals to corporate users and transferring the ownership of outstanding securities from one investor to another makes any study in the direction of helping the maintenance of fair and orderly markets for securities a worthwhile effort.

For a long time, the going presumption was that the efforts directed toward preventing fraudulent, deceptive, and manipulative acts in the securities markets would protect the individual investors and the public interest. Since 1970, however, the drastic impacts of the financial and operational difficulties of the broker/dealers on the securities markets have been widely recognized.

The Securities Investor Protection Corporation (SIPC) has been established to provide limited insurance for the investors whose broker/dealers go bankrupt. Although a significant step, the creation of SIPC could not solve the problems. For one thing, SIPC operates on an after-the-fact basis. That is, when a member firm falls into financial trouble, SIPC initiates steps to have it placed in liquidation. Secondly, the protection provided by SIPC against financial loss to customers of the failed firm is limited to $50,000 per customer, no more than $20,000 of which may be for claims for cash,[1] whereas there have been several cases in which claims for securities or cash were greater than these limits.[2] And most importantly, SIPC can do nothing about the misallocation of scarce resources embraced in each failure and wastage of time, money, and efforts involved in each liquidation.

It is desirable, therefore, to contrive a means by which the firm which is approaching a financial or operational difficulty be identified at the earliest time possible, so that opportunities can be provided for corrective actions and potential detriments can be avoided.

The present study is an attempt in this direction. More specifically, the purposes of this dissertation are: (1) to identify potential ecomomic and non-economic factors which might differentiate between financially-distressed firms and those which appear to be economically viable; (2) to statistically determine a set of variables which has the highest discriminatory power; and (3) to suggest both public and managerial action which might be taken in light of the statistical results obtained in point 2.

To ensure these purposes, an analytic framework was set forth in Chapter II, where it was hypothesized that the probability of financial distress of a firm at time t is a function of the net cash-flow per dollar of assets at time t, extent of the risk exposure, and the mismanagement of the firm. It was also concluded that, because specific relations between these elements and the probability of financial distress could not be known, the proposed general relation should be expanded to include all possible factors affecting the above-mentioned elements.

In that chapter, three potential approaches for predicting the financial position of a firm in the near future were discussed. They were: "deterministic," probabilistic, and blend approaches. The "deterministic" approach was meant to illustrate the traditional method of prediction using the historical financial ratios in conjunction with the MDA technique. The probabilistic approach was stated to be the one which does not treat the financial ratios and other related forces affecting the firm's financial positions as factors whose values can be determined with certainty at any point of time in the near future. Rather it attaches some probability distribution to each factor, based upon its historical performance. And, instead of using the MDA technique, it relies upon the modern probability theories.

The probabilistic approach suggested in this study, although intriguing, requires a great deal of research before it can be actually put to work. The blend approach, used in the present study, was therefore proposed to act as a bridge between the traditional and the probabilistic approaches. This approach, while retaining all the favorable properties of the "deterministic approach, especially with respect to the utilization of the MDA technique, incorporates some facets of the probability approach. More specifically, it introduces and utilizes variables which measure the volatility and variations of most of the significant internal and external factors having impacts on the firm's performance.

Thus, in this way, most of the potential economic and non-economic factors, which might differentiate between failed and nonfailed firms, were identified. But, before these factors were exposed to the MDA technique, this technique was previewed in Chapter III to provide the required groundwork for the empirical testing of the hypothesis.

Chapter III presented an overall picture of the MDA technique in application in a summary form. The related mathematical formulations were mentioned only up to a point where they have direct bearing on the understanding of the application. A section of the historical development, however, provided sufficient references for further information. A discussion of the general assumptions gives an idea about the conditions under which the results are more reliable. Relation to regression analysis, relative importance of variables, reliability tests, and the available MDA computer programs were the other topics discussed in Chapter III. The chapter concludes after a brief survey of the application of MDA in the financial area.

The next step was to empirically test the suggested hypothesis. But, to fully test the hypothesis, there was a need to have data on the most significant factors included in the expanded form of the hypothesized function. Unfortunately, however, the efforts to obtain such data proved to be futile. The Public Reference Office of the SEC, although cooperative in supplying the financial Forms X-17A-5 of the selected firms, denied the accessibility of Forms X-17A-10, on the grounds that they were confidential.

As a consequence of the above problem, reconsideration of all pertinent factors, substitution of some variables, and modification of the hypothesized function became an absolute necessity. In particular, the cash-flow element in the function was substituted with a more general term, the profitablity.

To conduct the empirical test, 60 SIPC failed firms and 60 nonfailed firms were selected and each of them was randomly divided into two equal-size groups. One group was used in an original sample, and the other in a holdout sample. The original sample was used to derive the discriminant function and the holdout sample to test the validity and predictability of the derived function. These samples were used in several experiments, two of which constituted Test I and Test II, and the rest were utilized for evaluating the potential biases in the original discriminating model, Test I.

Test I is the main experiment which has been conducted to analyze the financial position of the firms. This test is based upon the last financial reports of the failed firms and the pertinent reports of the nonfailed firms. Since the mean value of the last report lag time is about 11.5 months, the model's predictive span may be construed as being about one year.

In this test, a total of 83 variables was examined and 12 of them were selected as an "optimal" subset by a multi-stage stepwise selection procedure. These variables measure the profitability, risk exposure, and mismanagement of a firm. The model which incorporates these 12 variables separates the failed and nonfailed groups in such a way that there is virtually no overlap between them and the distance between the group means is highly signficant.

The model was very helpful in describing the attributes of the failed and nonfailed firms. The sample firms showed the association of lower ages of a firm and of its president with failure. They also demonstrated that nonfailed firms, on average, have relatively greater retained earnings to total assets. The interest rates in the prior year have also had positive effects on the nonfailed firms.

The relations between failure and percentage changes in equity capital, percent of the short positions in the trading and investment accounts, and the coefficient of variation of the ratio of the trading and investment accounts to total assets were shown to be positive. This direct relation also existed for variables dealing with partly secured and unsecured customers' securities accounts, total liabilities minus subordinated debt, and property and fixture management.

The accuracy and validity of the model were examined by several methods. The quadratic classification procedure misclassified only 1.67 percent of the observations in the original sample. This very low percentage of misclassification indicates that the model is significantly accurate. The expected rate of misclassification in the holdout sample was 13.3 percent. This rate also reveals that the discriminatory power of the model is satisfactory. The scrambled method, the Lachenbruch classification procedure, and the "normalized" model did not indicate that the potential biases in the model are significant.

In an attempt to examine the possibility of the extension of the inferential and predictive use of the MDA technique to two years prior to the bankruptcy of securities brokerage/dealer firms, Test II was conducted. This test also utilized the quadratic discriminant model with 12 variables. Based upon the information contained in the financial reports issued two years prior to bankruptcy, the model constructed in this test was able to reclassify 96.7 percent of the observations in the original sample correctly. The expected rate of misclassification in the holdout sample was not, however, as good as that of Test I. This rate was 33.3 percent. Although this rate is high, it is significantly better than pure chance. The result, however, was not unexpected because the specific nature of the securities industry and the kinds of the available data make any remote prediction difficult, if not impossible.

Conclusion

The relatively low rates of misclassification both in the original sample and holdout sample of Test I along with the results of H_2 test and group overlap examination indicate that:

- The failed securities brokerage/dealer firms possess some attributes which are different from those of the nonfailed firms.
- These attributes can be identified by the blend approach which incorporates some probabilistic variables, even with such a scant amount of financial data as those available to the present study. And,
- The attributes can be utilized in predicting the failure for a period as long as about one year prior to bankruptcy.

Furthermore, the relative discriminatory power of the selected variables show that the risk element is the most significant determinant of the firm's survival. The importance of profitability is next and that of the mismanagement is last.

The results of Test II were not as good as Test I. This was not unexpected, however, and may signify that the more remote span of the prediction weakens the predictive power of the model, and for obtaining better results there is an absolute need for a more complete set of data than what was available to the present study.

Implications

The results obtained from this work indicate that the blend approach applied to the empirical experiment is a useful method for dealing with predictive issues of this nature. A model of the type developed in this study can be readily used by all the parties having interests in a securities brokerage/dealer firm.

Since the enactment of the Securities Act of 1933, the securities industry has been increasingly put under the scrutiny and control of numerous governmental regulatory and self-regulatory organizations. The numerous and ever-increasing rules and regulations; the various forms, reports, and paperwork requirements; and the overlapping and occasionally conflicting functions of these agencies have created a situation which is fully reflected in the related studies reviewed in Chapter I and is partially indicated by the words of the Subcommittee on Securities when it says:

> ...Regulation, by government and by industry groups, is an essential element in protection of investors, but is not an effective substitute for competition in assuring a flexible and healthy industry....[3]

Under such a circumstances, if and/or when a model such as the one employed in this study can be developed in the best way, some of these problems may be solved. By using such a model, the management of a firm may monitorize itself before the situation becomes irreversible. The creditor may effectively evaluate the firm's positions before feeling sorry. The investor may take a right step before losing his capital. And the free-market principle may become operative before it collapses.

All these can be accomplished only if the information upon which the model is constructed is accurate, suitable, reliable, and available to the public. As emphasized by the Subcommittee on Securities in its Securities Industry Study, a high order of importance should be accorded to the public disclosure of the financial condition of brokers and dealers. In the Subcommittee's words:

> ...The importance of public disclosure of information regarding the financial condition of brokers and dealers should not be underestimated....[4]

In view of the above comments, it seems that the public's interest is better served if efforts are focused on creating conditions which lead to the provision of accurate, suitable, reliable, and accessible financial data of the securities brokerage/dealer firms.

Suggestions for Further Research

Based upon the results of this study, two areas seem to deserve further research. The first is short-run and is more practical. The second one is more suitable for a long-range endeavor and is theoretical in nature.

First Area

This area may be considered as application of the analytic framework set forth in Chapter II to a more complete set of data than what was available to the present study.

Second Area

This area may be considered as the consecutive steps leading to the development of the probabilistic approach to prediction, as discussed in Chapter I. More specifically, the research needed comprises:

(1) Identifying all the probabilistic factors affecting the probability of the firm's financial distress.

(2) Determining the inter-relations between these factors.

(3) Finding a suitable way to estimate the *a priori* probabilities of such factors, and finally

(4) Formulating the probabilistic model.

NOTES

1. Recently, it has been proposed that the above-mentioned limit be raised to $100,000, of which no more than $40,000 may be for claims for cash. But, even when the proposal is enacted, it is expected that the uncovered cases would still remain in a considerable number. This would be due to both inflation and the higher level of the accumulated wealths.

2. See SIPC Fourth Annual Report 1974, p. 2

3. See Note 7 of Chapter I.

4. U.S. Senate, Securities Industry Study, Report of the Subcommittee on Securities, Committee on Banking, Housing, and Urban Affairs (Committee Print), Washington, D.C., U.S. Government Printing Office, February, 1973, p. 39.

APPENDIX A

FAILED FIRMS IN THE ORIGINAL SAMPLE

No.	Name of the Firm	Age	No. of Reports	LRLT (in months)[a]	Z-score[b]	Group Assignment[b,c]
1	Financial House, Inc.	18	3	13	-6514.82	
2	Equitable Equities, Inc.*	2	2	10	-33.71	
3	Pacific Western Securities*	7	3	5	-235.50	
4	R.S. Emerson Co.	5	3	5	-265.25	
5	Smith & Medford, Inc.	3	3	12	-67.63	
6	Hill, Curtin & Ackroyd, Inc.*	3	2	16	-39.65	
7	First Midwest Investment Corp.*	4	3	2	-473.03	
8	Media Financial Services	18	3	26	-44.88	
9	Universal Underwriting Service, Inc.	3	3	14	-980.33	
10	S.J. Salmon & Co., Inc.	4	3	14	-66.82	
11	Aberdeen Securities Co., Inc.	2	2	9	-51.47	
12	C.H. Wagner & Co., Inc.*	3	2	14	-1532.40	
13	John E. Samuel & Co.*	10	3	3	-255.70	
14	Morgan, Kennedy & Co., Inc.	7	3	21	-25.30	
15	N.F. James & Co., Inc.	2	2	18	-1187.92	
16	Havener Securities Corp.	13	3	14	-219.48	
17	Klee & Co., Inc.	2	2	8	-112.63	
18	Doores Securities Corp.	2	2	13	-258.86	
19	Northeast Investors Planning Corp.	3	2	10	-139.16	
20	Bovers, Parnass & Turel, Inc.	4	3	7	-41.32	
21	A.J. Orsino Securities, Inc.	4	3	11	-26.81	
22	Kelly, Andrews & Bradley, Inc.*	3	3	7	-824.52	
23	Parker, Jackson & Co.*	11	2	9	-787.06	
24	Packer, Wilbur & Co., Inc.*	10	2	17	-112.16	
25	Stewart Securities Corp.	16	3	6	-10072.55	
26	Orin R. Dudley	8	2	14	-56.28	

No.	Name of the Firm	Age	No. of Reports	LRLT (in months)[a]	Z-score[b]	Group Assignment[b,c]
27	Howard Lawrence Co.*	5	2	15	-163.06	
28	Harper Johnson Co., Inc.	4	3	19	-11.05	
29	First Continental Securities, Inc.	8	3	16	-65.13	
30	Equidyne*	2	2	14	-21051.97	

[a] LRLT indicates the number of months elapsed between the date of the last financial report and the filing date of the SIPC application in court.

[b] Figures in this column are related to Test 1.

[c] Only the misclassified cases are specified.

*Firms omitted for the test of possible biases due to the missing data.

APPENDIX B

NON-FAILED FIRMS IN THE ORIGINAL SAMPLE

No.	Name of Firm	Age	Z-Score[a]	Group Assignment[a,b]
1	Holt Securities, Inc.	NA	5243.58	
2	Beyer & Co.	27	40.64	
3	Emch & Co.	29	76.84	
4	D.W. Daniel Co., Inc.*	14	56.74	
5	Robert M. Mitchell	NA	124.76	
6	Auerbach, Pollak & Richardson, Inc.	6	1947.17	
7	Financial Investors of Westchester	4	19.71	
8	Gary Ishikawa & Co.	10	50.76	
9	Ginberg & Co., Inc.	12	9.64	
10	Fourdee Planning Corp.*	7	53.03	
11	Pitfield, Mackay & Co., Inc.	35	60.33	
12	Frank Knowlton & Co.*	21	22.95	
13	Penn Square Management Corp.*	4	3974.55	
14	Lasker, Stone & Stern	25	388.15	
15	Prevision Investment Co.	10	16.74	
16	Pension Equities Co.	NA	85.80	
17	First Midstate Incorporated	8	10.02	
18	Balanced Investment, Inc.*	10	81.91	
19	The Daiwa Securities Co., Inc.	6	37479.12	
20	M. Rogers Shailer	7	80.42	
21	Rowe & Pitman, Inc.	4	12.31	
22	Wagenseller & Durst, Inc.	24	80.23	
23	Tri-Line Planning Corp.*	11	58.86	
24	Harold C. Brown & Co., Inc.	19	73.50	
25	Finance Securities, Inc.	7	28.23	
26	Engeler & Budd Co.	11	-6.89	Failed

No.	Name of the Firm	Age	Z-Score[a]	Group Assignment[a,b]
27	Frederick & Co., Inc.	11	6.48	
28	James I. Black & Co.	8	211.40	
29	Burgan Investment & Sec.	9	514.45	
30	Carolina Investors Corp.	7	57.81	

[a] Figures in this column are related to Test 1.
[b] Only the misclassified are specified.

NA indicates data not available.

Note: As compared with Appendix A, Appendix B omits the column on "No. of Reports" because the number of reports for all firms has been 3. The LRLT column has also been omitted because it was not relevant.

*Firms omitted for the test of possible biases due to the missing data.

APPENDIX C

FAILED FIRMS IN THE HOLDOUT SAMPLE

No.	Name of the Firm	Age	No. of Reports	LRLT (in months)^{a}	Z-score^{b}	Group Assignment^{b,c}
1	Albert & Maguire Securities Co.*	4	2	6	-1859.73	
2	Carlton Cambrige & Co., Inc.	6	3	16	-13.09	
3	Security Planning, Inc.*	7	3	19	-22298.14	
4	Lexington Capital Corp.*	4	2	8	-259.98	
5	Glendale Securities Corp.	3	3	6	-227.78	
6	J. Shapiro Co.	5	3	10	-434.13	
7	C.I. Oren & Co.	4	3	16	-70.07	
8	Custodian Security Brokerage Corp.	2	2	11	-143.00	
9	Ambassador Church Finance Development Group	4	3	11	-49.07	
10	Security Planners, Ltd., Inc.	3	2	3	-689.49	
11	Baron & Co., Inc.	3	3	2	-248.80	
12	International Funding Securities, Inc.*	9	2	21	-68.48	
13	Schreiber Bosse & Co., Inc.	4	2	6	-20.58	
14	Kenneth Bove & Co., Inc.	6	3	6	-103.36	
15	J.R. Narwitz & Co.	5	3	32	-5.97	
16	P.L.M. Securities, Inc.	4	2	16	-19317.40	
17	Joseph Garofalo Co.	3	2	12	-2650.04	
18	Holt, Murdock Securities, Inc.	3	3	5	-609.50	
19	G.M. Stanley & Co., Inc.*	3	3	7	-52.03	
20	Forma Securities, Inc.	4	3	15	-6831.98	
21	Karle R. Berglund*	3	2	27	1378.29	Nonfailed
22.	P&H Associates*	3	3	5	-888.92	
23	Llorens Associates, Inc.*	4	3	7	-742.27	
24	First Eastern Investment Corp.	14	3	12	-193.19	
25	First Investment Savings Corp.*	15	3	6	-34.42	

No.	Name of the Firm	Age	No. of Reports	LRLT (in months)[a]	Z-Score[b]	Group Assignment[b,c]
26	McMahon & Hoban, Inc.*	34	3	4	51.94	Nonfailed
27	Stan Ingram & Associate	3	2	23	45.62	Nonfailed
28	Commonwealth Securities Corp.*	9	3	2	-18558.61	
29	Mid-Continent Securities Co., Inc.	22	3	4	30.30	Nonfailed
30	Frank & Drake, Inc.	4	3	14	-58.91	

[a] LRLT indicates the number of months between the date of the last financial report and the filing date of the SIPC application in court.

[b] Figures in this column are related to Test 1.

[c] Only the misclassified cases are specified.

*Firms omitted for the test of possible biases due to the missing data.

APPENDIX D

NONFAILED FIRMS IN THE HOLDOUT SAMPLE

No.	Name of the Firm	Age	Z-Score[a]	Group Assignment[a,b]
1	A.E. Ames & Co.	37	11766.96	
2	Drysdale & Co.	37	5121.75	
3	C.A. Botzum & Co.	9	7300.86	
4	Country Capital Management Co.	7	37.31	
5	Commonwealth Investment Planning	6	76.97	
6	Investment Securities Corp.	5	241.95	
7	S.A. Judah & Co., Inc.	3	29.61	
8	Eastern Securities, Inc.	18	128.20	
9	Investment Co., Ltd.	14	37.61	
10	Hettleman & Co.	7	1209.60	
11	W.H. Newbold's Son & Co.	18	176.37	
12	J. Morgenstern & Co.	11	-1961.09	Failed
13	H. Kawano & Co., Inc.	14	-37.55	Failed
14	E. Lowitz & Co.	33	498.80	
15	Oftring & Co., Inc.	10	6.17	
16	Robert M. Horne	11	97.17	
17	Hackett & Co., Inc.	9	43.43	
18	Doft & Co., Inc.	11	90.28	
19	Wildman, Neal & DeBolt, Inc.	8	72475.47	
20	David H. Rittmaster & Co.	13	32.39	
21	Thomas, Haab & Botts	31	-0.12	
22	Reger & Co., Inc.*	11	6.82	
23	Allen, Rogers & Co., Inc.*	6	9.16	
24	Wurzburger, Morrow & Keouch, Inc.*	18	59.78	
25	Conners & Co., Inc.	11	-7.86	Failed
26	Murch & Co., Inc.	14	14.42	

No.	Name of the Firm	Age	Z-Score[a]	Group Assignment[a,b]
27	Manger-Fisher-Lona, Inc.	10	67.52	Failed
28	Imperial Securities, Inc.	6	-278.87	
29	C&M Investment Center, Inc.*	10	23.06	
30	E.L. Aaron & Co., Inc.	11	10.11	

[a] Figures in this column are related to Test I.
[b] Only the misclassified cases are specified.

Note: As compared with Appendix C, Appendix D omits the column on "No. of Reports" because the number of reports for all firms has been 3. The LRLT column has also been omitted because it was not relevant.

*Firms omitted for the test of possible biases due to the missing data.

APPENDIX E

A LIST OF DATA SET ITEMS

President's age.

President's educational background (coded 0 for under high school, 1 for high school, 2 for junior college, 3 for a bachelor's degree, 4 for a master's degree, and 5 for a doctoral degree).

President's experience in general business (other than securities business) during past ten years.

President's experience in securities business during past ten years.

Firm's age in years.

Last Report Lag Time (LRLT) in months.

Date of the last report.

Number of reports.

Date of the subject report.

Total assets.

Cash on hand and in banks plus certificate of deposits.

Total accounts receivable.

- Accounts receivable from broker/dealers (total).
    - Failed to deliver (total).
        - Failed to deliver within 30 days
        - Failed to deliver over 30 days
    - Other broker/dealers accounts with debit balances.
- Accounts receivable from customers (see Question 6 of the Financial Questionnaire, Form X-17A-5).
    - Cash accounts (see Question 6A.1 of the form above).
    - Secured accounts (see Question 6B.1 of the form above).
    - Partly secured accounts (see Question 6C.1 of the for above).
    - Unsecured accounts (see Question 6D above).

Accounts receivable from directors and officers

Accounts with debit balances on trading and investment.

Accounts with credit balances on trading and investment.

Property, furniture, fixtures, and leasehold.

Deposits other than the certificate type.

Exchange membership value.

Money borrowed, secured by or containing customer's collateral.

Money borrowed, secured by the firm's collateral.

Total accounts payable.

Accounts payable to broker/dealers.

Failed to receive.

Failed to receive within 30 days.
Failed to receive over 30 days.

Other broker/dealers accounts with credit balances.

Accounts payable to customers.
Accounts payable to directors and officers.

Type of the organization (coded 0 for corporation, 1 for partnership, and 2 for sole proprietorship).

Total capital account.

Stocks
Treasury stocks
Paid-in surplus
Retained earnings.

Subordinated debt.

Commission accounts with debit balances.

Commission accounts with credit balances.

Time period between two consecutive reports.

Population of the city where the firm's main business is located.

Per capita income of the city mentioned above.

GNP in the reporting year period.

Dow-Jones Total Index (annual averages of monthly figures).

Standard and Poor's Common Index (annual averages of monthly figures).

Interest rates (bank rates on short-term business loans)

Total market value of bond and stock sales in the reporting year.

APPENDIX F

LIST OF THE VARIABLES[†]

X-1 = Company's code number.
X-2 = President's educational background.
X-3 = President's experience in general busines (years).
X-4 = President's experience in securities business (years).
X-5 = Firm's age.
X-6 = President's age.
X-7 = Cash/TA*
X-8 = (Cash + Other Deposits + Membership)/TA*
X-9 = (Cash + Other Deposits)/TA.
X-10 = Retained Earnings/Equity Capital.
X-11 = Retained Earnings/(Equity Capital + Subordinated Debt).
X-12 = Retained Earnings/TA.
X-13 = Equity Capital/TA*
X-14 = Total Liabilities/Equity Capital.
X-15 = (Equity Capital + Subordinated Debt)/TA.
X-16 = (Total Liabilities - Subordinated Debt)/Equity Capital*
X-17 = Total Liabilities - Subordinated Debt)/(Equity Capital + Subordinated Debt).
X-18 = (Total Liabilities - Subordinated Debt)/TA.
X-19 = Total Liabilities/TA.
X-20 = Subordinated Debt/(Equity Capital + Subordinated Debt).
X-21 = AC Receivable/TA.
X-22 = (Partly Secured AC + Unsecured AC)/TA. Partly Secured AC are customers' securities accounts liquidating to a deficit (Question 6,C of the Financial Questionnaire). Unsecured AC are customers' securities accounts which are reported under Question 6,D of the Questionnaire.
X-23 = AC Receiveable from BD/TA.
X-24 = AC with Debit Balances on Trading and Investment/TA*
X-25 = Failed to Deliver/TA*
X-26 = Failed to Deliver within 30 Days/TA*
X-27 = (Failed to Deliver + Failed to Receive)/TA.
X-28 = Failed to Deliver Outstanding 30 days or more/TA.
X-29 = (Failed to Deliver 30 Days or More + Failed to Receive 30 Days or More)/TA.
X-30 = AC Receivable from Customers/TA.
X-31 = Failed to Receive within 30 Days/TA*
X-32 = Customers' Securities Cash AC (Question 6,A)/TA*
X-33 = Customers' Securities Secured AC (Question 6,B)/TA.
X-34 = Customers' Securities Unsecured AC (Question 6,D)/TA*
X-35 = (Borrowings + AC Payable)/TA*

X-36 = (Borrowings + AC Payable + Subordinated Debt)/TA*
X-37 = Borrowings/TA.
X-38 = AC Payable to BD/TA.
X-39 = Failed to Receive Outstanding 30 days or More/TA*
X-40 = AC Payable to Customer/TA.
X-41 = (AC with Debit Balances on Trading and Investment + AC with Credit Balances on Trading and Investment)/TA.
X-42 = Δ(Total Liabilities - Subordinated Debt)/ΔTA (changes from the year before last).
X-43 = Δ(Total Liabilities - Subordinated Debt)/ΔEquity Capital (changes from the year before last.)
X-44 = Δ Cash/ΔTA (changes from the year before last).
X-45 = ΔEquity Capital/(Equity Capital)$_{t-1}$. The numerator is the change in equity capital from the year before last to the last year, and the denominator is the amount of the equity capital in the year before last.
X-46 = ΔEquity Capital/ΔTA (changes from the year before last)*
X-47 = ΔRetained Earnings/(TA)$_{t-1}$. (Similar explanation as for X-45.)
X-48 = ΔRetained Earnings/ΔTA (changes from the year before last).
X-49 = ΔGNP/(GNP)$_{t-1}$ (similar explanation as for X-45).
X-50 = $(R_1-d_1)/\sigma_1$, where R_1 is the firm's age, d_1 is the disaster level for the age factor which is the mean value of the ages of the failed firms in the original sample, and σ_1 is the standard deviation of the values whose mean is d_1.++*
X-51 = $(R_2-d_2)/\sigma_2$, where R_2 is the age of the firm's president, d_2 is the disaster level for the age factor which is the mean value of the ages of the presidents of the failed firms in the original sample, and σ_2 is the standard deviation of the values whose mean is d_2.++*
X-52 = AC with Credit Balances on Trading and Investment/(AC with Debit Balances on Trading and Investment + AC with Credit Balances on Trading and Investment).
X-53 = Property and Fixtures (Question 13 of the Questionnaire)/TA.
X-54 = GNP in the reporting year.
X-55 = Dow-Jones Total Index (annual averages of monthly figures).*
X-56 = Standard and Poor's Common Index (annual averages of monthly figures).*
X-57 = (Interest Rate)$_t$, where t stands for the firm's last-report year.
X-58 = (Interest Rate)$_{t-1}$, where t stands for the firm's last-report year.

X-59 = Total Market Value of Bond and Stock Sales (in the reporting year and in \$ million)*

X-60 = Log_{10} of the Firm's City Population.

X-61 = Per Capita Income on Firm's City.

X-62 = Failed to Receive/TA*

X-63 = Coefficient of Variation of X-7.

X-64 = Coefficient of Variation of X-14.

X-65 = Coefficient of Variation of X-15.

X-66 = Coefficient of Variation of X-12.

X-67 = Coefficient of Variation of X-25.

X-68 = Coefficient of Variation of X-22.

X-69 = Coefficient of Variation of X-41.

X-70 = Coefficient of Variation of X-52.

X-71 = Coefficient of Variation of X-53.

X-72 = Coefficient of Variation of X-62.

X-73 = $(R_3-d_3)/\sigma_3$, where R_3 and σ_3 are, respectively, mean and standard deviation of X-7 and d_3 is the disaster level for X-50 which is calculated by averaging the values of X-50 in the failed firms group of the original sample.††

X-74 = $(R_4-d_4)/\sigma_4$ (similar explanation as in X-73, but for X-14)*

X-75 = $(R_5-d_5)/\sigma_5$ (similar explanation as in X-73, but for X-15).

X-76 = $(R_6-d_6)/\sigma_6$ (similar explanation as in X-73, but for X-12).

X-77 = $(R_7-d_7)/\sigma_7$ (similar explanation as in X-73, but for X-25).

X-78 = $(R_8-d_8)/\sigma_8$ (similar explanation as in X-73, but for X-22).

X-79 = $(R_9-d_9)/\sigma_9$ (similar explanation as in X-73, but for X-41).

X-80 = $(R_{10}-d_{10})/\sigma_{10}$ (similar explanation as in X-73, but for X-52).

X-81 = $(R_{11}-d_{11})/\sigma_{11}$ (similar explanation as in X-73, but for X-53).

X-82 = $(R_{12}-d_{12})/\sigma_{12}$ (similar explanation as in X-73, but for X-62).

X-83 = Total Assets (a proxy for the firm's size).

X-84 = Δ (Standard and Poor's Common Index)/(Standard and Poor's Common $\text{Index})_{t-1}$, where the nominator is the change from year t-1 to year t, to show the result as the percentage change*

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†For the data items used in constructing these variables, see Appendix E. Meanwhile, the following abbreviations and symbols have been used in the variable list, for simplification:

AC for Accounts.
BD for Broker/dealers.
TA for Total Assets.
Δ for changes in.
/ for division.

[++] For analytical discussion on this variable, see the subsection on "variable set" in Test I.

*These variables were later omitted from the analysis due to their high correlations with other variables (see Appendix G).

APPENDIX G

THE TOTAL CORRELATION MATRIX OF TEST I

```
ROW         X_2             X_3             X_4             X_5             X_6             X_7             X_8             X_9
 X_2   1.000000       -0.7090735E-01  -0.1979971      -0.5726144E-01  -0.2861871      -0.2407377      -0.2388121      -0.2394778
 X_3  -0.7090735E-01   1.000000       -0.7182550      -0.2045017      -0.1246834       0.9807229E-01   0.7361615E-01   0.8691370E-01
 X_4  -0.1979971      -0.7182550       1.000000        0.2420440       0.2423386       0.1948598       0.2253044       0.2097816
 X_5  -0.5726144E-01  -0.2045017       0.2420440       1.000000        0.4090801       0.1401873E-01   0.5029820E-01   0.3714371E-01
 X_6  -0.2861871      -0.1246834       0.2423386       0.4090801       1.000000        0.1722156       0.2001101       0.1979147
 X_7  -0.2407377       0.9807229E-01   0.1948598       0.1401873E-01   0.1722156       1.000000        0.9899154       0.9943931
 X_8  -0.2388121       0.7361615E-01   0.2253044       0.5029820E-01   0.2001101       0.9899154       1.000000        0.9943202
 X_9  -0.2394778       0.8691370E-01   0.2097816       0.3714371E-01   0.1979147       0.9943931       0.9943202       1.000000
X_10  -0.9451646E-02  -0.7023144E-01   0.2381925E-01   0.2817016E-01  -0.2088345E-01  -0.4714641E-01  -0.3211323E-01  -0.3937913E-01
X_11  -0.1452289      -0.1827941       0.1309371       0.2513731       0.1821281       0.1997211E-01   0.2458384E-01   0.2266053E-01
X_12  -0.1472355      -0.1927868       0.1745782       0.3546489       0.3291327       0.2082263       0.2142816       0.2161469
X_13  -0.2745759       0.4358798E-01   0.8149180E-01   0.1164472       0.3768629       0.4108809       0.3886988       0.4061661
X_14  -0.4515571E-01   0.1872222      -0.2120890      -0.7262611E-01  -0.8545655E-01  -0.4479056E-01  -0.4483158E-01  -0.3463891E-01
X_15  -0.2657499       0.4496638E-01   0.8005571E-01   0.6350893E-01   0.2931454       0.3509674       0.3412361       0.3364293
X_16   0.4220176E-02   0.1721885      -0.2057410      -0.1141046      -0.1444717      -0.1092800      -0.1065473      -0.9952993E-01
X_17   0.1762883       0.9229910E-01  -0.1168321      -0.6352639E-01  -0.1214005      -0.7643956E-01  -0.7022607E-01  -0.6844378E-01
X_18   0.2657574      -0.4496301E-01  -0.8008128E-01  -0.6350672E-01  -0.2931347      -0.3509640      -0.3412322      -0.3364259
X_19   0.2745830      -0.4358533E-01  -0.8148298E-01  -0.1164457      -0.3768511      -0.4108794      -0.3886962      -0.4061646
X_20   0.8976233E-01   0.3755414E-01  -0.1094417      -0.1758823      -0.2829822      -0.2413732      -0.2281246      -0.2595692
X_21   0.2217642      -0.7964456E-01   0.1868888E-01  -0.4234426E-01  -0.1322206      -0.4539645      -0.4367148      -0.4317947
X_22   0.6231972E-01   0.2334987      -0.3520988      -0.7763876E-01  -0.1664945      -0.2061512      -0.2413963      -0.2319926
X_23   0.8972341E-01  -0.9562320E-01   0.8022016E-01  -0.1174420      -0.1684219      -0.3145120      -0.3111688      -0.3100212
X_24  -0.2109570E-01  -0.7115686E-01  -0.8015306E-01   0.9395659E-01   0.1506233      -0.3231187      -0.3607526      -0.3318214
X_25   0.1631694      -0.2882959E-01   0.1661478E-01  -0.2585399      -0.2429309      -0.2623509      -0.2704169      -0.2531282
X_26   0.1667579      -0.4001145E-01   0.2715874E-01  -0.2864978      -0.2267261      -0.2552533      -0.2630893      -0.2577726
X_27   0.1544741      -0.9698325E-01   0.9214509E-01  -0.2398199      -0.2383033      -0.1407158      -0.1523245      -0.1367845
X_28   0.7641894E-01   0.1029641      -0.1049371      -0.3230622      -0.3513138      -0.2649192      -0.2665112      -0.2575694
X_29   0.1082098       0.2030614      -0.1226879      -0.3390155      -0.4288583       0.2217744E-01   0.7320959E-02   0.1836755E-01
X_30   0.2695318       0.3485779E-01  -0.1141603       0.1781729      -0.2297179E-01  -0.3061768      -0.2803078      -0.2775338
X_31   0.1060286      -0.1945469       0.1784021      -0.9632075E-01  -0.1311103      -0.2676632E-01  -0.3655479E-01  -0.2077024E-01
X_32   0.1742694      -0.7317178E-01  -0.2735770E-01   0.2431636       0.6491464E-01  -0.2502134      -0.2267069      -0.2250948
X_33   0.3207013       0.9596235E-01  -0.5006743E-01   0.5079460E-01  -0.8025938E-01  -0.1781987      -0.1646879      -0.1691079
X_34   0.6555960E-02   0.2708179      -0.3737510      -0.3877131E-01  -0.1453336      -0.2469069      -0.2229329      -0.2149140
X_35   0.2815528      -0.6987131E-01  -0.6946248E-01  -0.3683837E-01  -0.2044694      -0.3093316      -0.2972327      -0.2936516
X_36   0.2897823      -0.6467193E-01  -0.7523036E-01  -0.9564519E-01  -0.3064889      -0.3796983      -0.1552466      -0.3741503
X_37   0.2041711       0.5795674E-01  -0.2348233       0.5870248E-01  -0.2064694E-01  -0.2299659      -0.2052802      -0.2231309
X_38   0.1867546      -0.1198370       0.6983912E-01  -0.8595844E-01  -0.2321488      -0.1037359      -0.1265858      -0.1078227
X_39   0.9782833E-01   0.2013806      -0.1029638      -0.2693918      -0.3661395       0.1434708       0.1261423       0.1356165
X_40   0.2666365E-01   0.8333832E-02  -0.5434539E-02  -0.1469216E-01  -0.1511814E-01  -0.1966609      -0.1594283      -0.1649157
X_41  -0.1034172E-01  -0.6655365E-01  -0.6703917E-01   0.6806573E-01   0.1223845      -0.3404320      -0.3783494      -0.3480687
X-42   0.2297348      -0.1790110       0.1698317      -0.9979135E-01  -0.1195144E-01   0.9159192E-04   0.3498712E-02  -0.1249633E-01
X-43   0.3439179E-01  -0.1179205       0.8117127E-01   0.1163706E-01  -0.1393799      -0.1662459      -0.1684272      -0.1664681
X-44  -0.3014880       0.1978006      -0.1955109      -0.1007032      -0.1330045       0.7765677E-02   0.7629473E-02   0.2373641E-01
X-45   0.2203643       0.1032822      -0.3286453      -0.1478206      -0.9136891E-01  -0.1849310      -0.2006766      -0.1991271
X-46  -0.2283281       0.1971023      -0.2323161      -0.7184774E-01  -0.1000368      -0.1010461      -0.9921229E-01  -0.6176321E-01
X-47   0.1875703      -0.1094345E-01  -0.2642525      -0.4647717E-02   0.4917588E-01  -0.1308094      -0.1502427      -0.1463454
X-48  -0.2460437       0.5632748E-01  -0.1209147       0.1661891E-02  -0.1476203E-01  -0.1049177      -0.9734846E-01  -0.8623114E-01
X-49  -0.8800459E-01   0.5685287E-01  -0.2709332E-02   0.1363960       0.5672201E-01  -0.3552888E-01  -0.4732269E-01  -0.1694330E-01
X-50   0.5726286E-01   0.2045043      -0.2420432      -1.000000       -0.4093831      -0.1401927E-01  -0.5029796E-01  -0.3714358E-01
X-51   0.2061826       0.1246886      -0.2423292      -0.4090779      -1.000005       -0.1722172      -0.2001113      -0.1979161
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TOT CORR SECTION   2

ROW         X_10           X_11            X_12            X_13            X_14            X_15            X_16            X_17
 X_2  -0.9451646E-02 -0.1452289     -0.1472355      -0.2745759      -0.4515571E-01 -0.2657499       0.4220176E-02  0.1752883
 X_3  -0.7023144E-01 -0.1827941     -0.1927868       0.4358798E-01   0.1872222       0.4496638E-01  0.1721885       0.9279910E-01
 X_4   0.2381925E-01  0.1309371      0.1745782       0.6349180E-01  -0.2120690       0.8009571E-01 -0.2057410      -0.1168321
 X_5   0.2817016E-01  0.2513731      0.3546489       0.1164472      -0.7262611E-01   0.6355393E-01 -0.1141048      -0.6352679E-01
 X_6  -0.2088345E-01  0.1821281      0.2291327       0.3768629      -0.8545655E-01   0.2931454     -0.1444717      -0.1214005
 X_7  -0.4714641E-01  0.1997211E-01  0.2082263       0.4108809      -0.4479356E-01   0.3509674     -0.1092809      -0.7643955E-01
 X_8  -0.3211323E-01  0.2458384E-01  0.2142816       0.3886988      -0.4483158E-01   0.3412361     -0.1065473      -0.7022607E-01
 X_9  -0.3937910E-01  0.2266053E-01  0.2161469       0.4061661      -0.3463891E-01   0.3364293     -0.9932003E-01  -0.6844379E-01
X_10   1.000000       0.3397203     -0.1224396      -0.1526859      -0.5915307       0.6580937E-01 -0.5830725      -0.4266210
X_11   0.3397203      1.000000       0.5213721       0.2074004      -0.2459655       0.6145052E-01 -0.3881220      -0.7802492
X_12  -0.1224396      0.5213721      1.000000        0.6008635       0.9673327E-01   0.3319664      0.4338439E-01  -0.4863363E-01
X_13  -0.1526859      0.2074004      0.6008635       1.000000       -0.2439287E-01   0.8164500     -0.1453886      -0.1526926
X_14  -0.5915307     -0.2459655      0.9673327E-01  -0.2439287E-01   1.000000       -0.2707207      0.9352366       0.4983472
X_15   0.6580937E-01  0.6145052E-01  0.3319664       0.8164500      -0.2707207       1.000000      -0.3002360      -0.1859832
X_16  -0.5830725     -0.3881220      0.4338439E-01  -0.1453886       0.9352366      -0.3002360      1.000000        0.6601459
X_17  -0.4266210     -0.7802492     -0.4863363E-01  -0.1526926       0.4983472      -0.1859832      0.6601459       1.000000
X_18  -0.6580877E-01 -0.6145045E-01 -0.3319653      -0.8164414       0.2707205      -0.9999921      0.3002360       0.1859838
X_19   0.1526859     -0.2074007     -0.6008639      -0.9999961       0.2439350E-01  -0.8164446      0.1453899       0.1526938
X_20   0.2970759     -0.3429968     -0.6000596      -0.6391227      -0.6253779E-01  -0.1261271      0.1081904       0.1201155
X_21  -0.1288459     -0.1050534     -0.2863050      -0.6014076       0.2876788      -0.7446887      0.3213364       0.2161363
X_22  -0.1366256E-01 -0.1079310     -0.1210545      -0.2884934       0.3708885      -0.3147570      0.4403704       0.1893820
X_23  -0.1327218     -0.1535717     -0.3053487      -0.4362719       0.1837586      -0.5032659      0.2135428       0.1955119
X_24   0.1907934      0.1524116      0.1837403       0.7651770E-01  -0.2330092       0.1573128     -0.1724375      -0.9167375E-01
X_25  -0.1385546     -0.1937267     -0.3789164      -0.4564554       0.1630946      -0.5053650      0.2230362       0.1852244
X_26  -0.1304299     -0.1878770     -0.3786209      -0.4617732       0.1715618      -0.5066662      0.2082964       0.1714166
X_27  -0.1501487     -0.1883634     -0.3372977      -0.5309501       0.2132823      -0.6625747      0.2565506       0.2201401
X_28  -0.1985703     -0.2614548     -0.2723712      -0.2656502       0.2577609      -0.3449860      0.3216679       0.2846777
X_29  -0.2139478     -0.3858082     -0.4567431      -0.3638356       0.1768836      -0.4198363      0.2120072       0.2347020
X_30  -0.6818396E-01  0.3827338E-01 -0.6765217E-01  -0.4273567       0.2336297      -0.5776442      0.2509583       0.1084788
X_31  -0.1082390     -0.8421808E-01 -0.1664083      -0.4765994       0.1974830      -0.6699908      0.2396707       0.1943505
X_32  -0.2522285E-01  0.1816759     -0.1261976E-01  -0.3871580       0.1286342      -0.5290194      0.1202610      -0.2978895E-01
X_33  -0.1139361     -0.1593630     -0.7585049E-01  -0.2268464       0.1562647      -0.3348022      0.1724250       0.2138239
X_34  -0.4818267E-01 -0.3157466E-01 -0.1074963      -0.2685257       0.3794543      -0.2846562      0.4467339       0.1975166
X_35  -0.4045388E-01 -0.2532604E-01 -0.2497907      -0.7635671       0.2138080      -0.9496095      0.2451215       0.1731223
X_36   0.1779227     -0.1805621     -0.5398809      -0.9651040      -0.2624165E-01  -0.7775381      0.9828490E-01  0.1425520
X_37   0.5241640E-01  0.9832704E-01 -0.3263212E-01  -0.1629705      -0.3127573E-01  -0.1993635     -0.6145680E-01  -0.5959724E-01
X_38  -0.1148273     -0.8134246E-01 -0.1492595      -0.5291957       0.2241769      -0.7308683      0.2617466       0.2283427
X_39  -0.1768311     -0.3534493     -0.4347879      -0.3249497       0.1011831      -0.3580936      0.1159184       0.1712499
X_40   0.5441998E-01 -0.3612987E-01 -0.2410911      -0.4927939       0.9361643E-01  -0.4820089      0.1307790       0.3239996E-01
X_41   0.1968991      0.1506322      0.1674191       0.4673104E-01  -0.2271584       0.1201484     -0.1675794      -0.6809972E-01
X-42  -0.5493648E-01 -0.2436708     -0.1011248E-01  -0.6187818E-01  -0.1773864      -0.6332111E-01 -0.1168932       0.3382432
X-43  -0.5485225E-02  0.5596345E-01  0.5930579E-01  -0.2012754       0.3343048      -0.2359088      0.4297354       0.2201607
X-44  -0.4302027E-01 -0.6535757E-01 -0.6964525E-01  -0.2870869E-01   0.2132236       0.2811956E-02  0.2576507       0.8337431E-02
X-45  -0.7355112E-01  0.6845150E-01  0.2343189       0.1466677       0.2742617E-01   0.8944160E-01 -0.1470661E-01   0.9523106E-02
X-46  -0.1264008      0.1210665      0.3357909E-01  -0.8167852E-02   0.3888664      -0.1559978E-01  0.4238223      -0.9476197E-01
X-47  -0.3236177E-01  0.1902233      0.4048584       0.2087166       0.2471504E-01   0.1603805     -0.5104292E-02  -0.1564523E-01
X-48  -0.8862448E-01  0.2186655      0.1180281       0.1814298E-02   0.3545402       0.1497798E-01  0.3848040      -0.1314225
X-49  -0.1226503     -0.8864349E-01 -0.2317266E-01  -0.1268988       0.3226452      -0.2730401      0.2978861       0.1985625
X-50  -0.2817239E-01 -0.2513720     -0.3546475      -0.1164460       0.7262552E-01  -0.6350803E-01  0.1141046       0.6352559E-01
X-61   0.2088361E-01 -0.1821250     -0.3291315      -0.3768544       0.8545554E-01  -0.2931414      0.1444698       0.1213981
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TOT CORR SECTION   3

ROW          X_18            X_19            X_20            X_21            X_22            X_23            X_24            X_25
 X_2     0.2657574       0.2745830       0.8976233E-01   0.2217642       0.6231972E-01   0.8972341E-01  -0.2179570E-01   0.1631694
 X_3    -0.4496301E-01  -0.4358533E-01   0.3755414E-01  -0.7964456E-01   0.2394987      -0.9562329E-01  -0.7115386E-01  -0.2892959E-01
 X_4    -0.8008128E-01  -0.8348298E-01  -0.1004417       0.1868888E-01  -0.3520988       0.8022016E-01  -0.8015308E-01   0.1661478E-01
 X_5    -0.6350672E-01  -0.1164457      -0.1758823      -0.4234426E-01  -0.7764876E-01  -0.1874420       0.9395659E-01  -0.2985399
 X_6    -0.2931347      -0.3768511      -0.2829822      -0.1332206      -0.1664945      -0.1684219       0.1506233      -0.2439339
 X_7    -0.3509640      -0.4108794      -0.2413332      -0.4539645      -0.2661512      -0.3145120      -0.3231187      -0.2623509
 X_8    -0.3412322      -0.3886962      -0.2281246      -0.4367148      -0.2418963      -0.3118688      -0.3637526      -0.2794169
 X_9    -0.3364259      -0.4061646      -0.2505692      -0.4317947      -0.2319926      -0.3100212      -0.3318214      -0.2593982
X_10    -0.6580877E-01   0.1526859       0.2970759      -0.1286459      -0.1366256E-01  -0.1327218       0.1997934      -0.1385546
X_11    -0.6145045E-01  -0.2074007      -0.3429968      -0.1050534      -0.1079310      -0.1535717       0.1524116      -0.1987267
X_12    -0.3312653      -0.6008639      -0.6000996      -0.2863950      -0.1210545      -0.3053487       0.1837403      -0.3739164
X_13    -0.8164414      -0.9999961      -0.6391227      -0.6014076      -0.2884934      -0.4362719       0.7651770E-01  -0.4564555
X_14     0.2707205       0.2439350E-01  -0.6253779E-01   0.2676788       0.3708485       0.1037586      -0.2330092       0.1839946
X_15    -0.9999921      -0.8164446      -0.1261271      -0.7446887      -0.3147570      -0.5032659       0.1573128      -0.5053659
X_16     0.3002360       0.1453899       0.1081904       0.3213364       0.4489704       0.2135428      -0.1724975       0.2230362
X_17     0.1859838       0.1526938       0.1201155       0.2161363       0.1679620       0.1855119      -0.9167975E-01   0.1652244
X_18     1.000001        0.8164507       0.1261279       0.7446929       0.3147572       0.5032682      -0.1573102       0.5353664
X_19     0.8164507       1.000000        0.6391242       0.6014113       0.2884947       0.4362743      -0.7651520E-01   0.4564584
X_20     0.1261279       0.6391242       1.000000        0.1163379       0.2230263       0.1107363       0.4910754E-01   0.1418331
X_21     0.7446928       0.6014113       0.1163379       1.000000        0.3816513       0.7745833      -0.2768673       0.6932769
X_22     0.3147572       0.2884947       0.2230263       0.3816513       1.000000        0.7492013E-02  -0.7703561E-01   0.3956967E-01
X_23     0.5032682       0.4362743       0.1107363       0.7745833       0.7492013E-02   1.000000       -0.2974717       0.9130306
X_24    -0.1573102      -0.7651520E-01   0.4910754E-01  -0.2768673      -0.7703561E-01  -0.2974717       1.000000       -0.2377878
X_25     0.5353664       0.4564584       0.1418331       0.6932769       0.3956967E-01   0.9130306      -0.2377878       1.000000
X_26     0.5066681       0.4617758       0.1470038       0.6943285       0.2201719E-01   0.9117046      -0.2395727       0.9989939
X_27     0.6625770       0.5309522       0.7228637E-01   0.6509635      -0.5466372E-02   0.6017884      -0.2143442       0.6910973
X_28     0.3449873       0.2656518       0.4316375E-01   0.4811921       0.2224674       0.6634867      -0.1603743       0.7316905
X_29     0.4198377       0.3638374       0.8742249E-01   0.2868941       0.6845814E-01   0.3806128      -0.1897873       0.4457873
X_30     0.5776455       0.4273590       0.2774912E-01   0.6031358       0.5984375      -0.1009357E-01  -0.1147460      -0.3415111E-01
X_31     0.6699910       0.4766024      -0.2525463E-01   0.4990511      -0.4806109E-01   0.5536989      -0.1312916       0.6244392
X_32     0.5290197       0.3871596      -0.1851781E-01   0.4871998       0.4015650      -0.4566940E-01  -0.4009574E-01  -0.5417220E-01
X_33     0.3348070       0.2268479      -0.1967664E-01   0.4324802       0.2541198       0.4687442E-01  -0.1684950      -0.1055260E-01
X_34     0.2866560       0.2685266       0.2310922       0.3695460       0.9579154       0.4566260E-01  -0.9271711E-01   0.6479019E-01
X_35     0.9496159       0.7635747       0.8785933E-01   0.6897489       0.2789117       0.4097789      -0.1653011       0.4381106
X_36     0.7775403       0.9651111       0.6182970       0.5577505       0.2601736       0.3602946      -0.8264458E-01   0.4027541
X_37     0.1993644       0.1629719      -0.2549076E-01   0.1310042       0.3333563      -0.1413397      -0.1739307      -0.1592263
X_38     0.7308701       0.5291994      -0.8028183E-02   0.4995980       0.4969975E-01   0.4274397      -0.4191045E-02   0.5003723
X_39     0.3580950       0.3249509       0.8699912E-01   0.1368190      -0.1468898E-01   0.1704004      -0.1610074       0.2195372
X_40     0.4826090       0.4927958       0.2412431       0.4139963       0.1121634       0.2740919      -0.1304803       0.2298895
X_41    -0.1201442      -0.4672924E-01   0.5115451E-01  -0.2398121      -0.7011348E-01  -0.2508891       0.9964027      -0.1856447
X-42     0.6332344E-01   0.6187961E-01  -0.9243637E-01   0.7727265E-01  -0.3619791       0.9981042E-01  -0.1519740       0.6997336E-01
X-43     0.2359095       0.2012767       0.1634465       0.2663269      -0.3067571E-01   0.2663195      -0.3244062E-01   0.2314817
X-44    -0.2811664E-02   0.2870885E-01   0.1518524      -0.8737868E-01   0.4065596      -0.5460739E-01   0.1879816E-01  -0.4213502E-01
X-45    -0.8943999E-01  -0.1466666      -0.1659158      -0.1130629       0.4348367E-02  -0.7876402E-01   0.2937566      -0.4322418E-01
X-46     0.1560049E-01   0.8166485E-02   0.1449246       0.4868248E-01   0.6312398      -0.6570771E-02  -0.1652752E-01   0.1927996E-01
X-47    -0.1603797      -0.2087161      -0.1540576      -0.1875166      -0.1622743E-01  -0.1732256       0.2663295      -0.1688401
X-48    -0.1497773E-01  -0.1814210E-02   0.1619246       0.3759594E-01   0.5924406      -0.1065699E-01  -0.1019236E-01   0.2787811E-02
X-49     0.2730470       0.1269035      -0.3062923E-01   0.3299834       0.2523428       0.2547959      -0.1739483       0.2430381
X-50     0.6350666E-01   0.1164451       0.1758803       0.4234380E-01   0.7764995E-01   0.1874409      -0.9395868E-01   0.2905391
X-51     0.2931387       0.3768536       0.2829795       0.1332209       0.1664944       0.1684211      -0.1506232       0.2429295
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TOT CORR SECTION    4

ROW         X_26            X_27            X_28            X_29            X_30            X_31            X_32            X_33
 X_2     0.1667579       0.1544741       0.7641894E-01   0.1082098       0.2695318       0.1068286       0.1742694       0.3297013
 X_3    -0.4001145E-01  -0.9698325E-01   0.1029641       0.2030614       0.3485779E-01  -0.1943469      -0.7917178E-01   0.9516235E-01
 X_4     0.2715874E-01   0.9214509E-01  -0.1049371      -0.1226879      -0.1141603       0.1784921      -0.2735770E-01  -0.5006743E-01
 X_5    -0.2884978      -0.2398199      -0.3230622      -0.3390155       0.1781729      -0.9622075E-01   0.2431636       0.5679460E-01
 X_6    -0.2267261      -0.2383033      -0.3513138      -0.4288583      -0.2237379E-01  -0.1313103       0.6431464E-01  -0.8025910E-01
 X_7    -0.2552533      -0.1407158      -0.2649192       0.2217744E-01  -0.3961769      -0.2076637E-01  -0.2502394      -0.1781987
 X_8    -0.2636893      -0.1520245      -0.2665112       0.7320959E-02  -0.2831078      -0.3655479E-01  -0.2287069      -0.1636979
 X_9    -0.2527726      -0.1367845      -0.2575604       0.1836755E-01  -0.2775318      -0.2977024E-01  -0.2230948      -0.1691979
X_10    -0.1304299      -0.1501487      -0.1885703      -0.2139438      -0.6818396E-01  -0.1082390      -0.2522285E-01  -0.1132361
X_11    -0.1878770      -0.1883634      -0.2614548      -0.3858082       0.3627338E-01  -0.8421908E-01   0.1816759      -0.1593730
X_12    -0.3786209      -0.1372977      -0.2723712      -0.4567431      -0.6765217E-01  -0.1664083      -0.1261976E-01  -0.7985949E-01
X_13    -0.4617732      -0.5309501      -0.2656502      -0.3638356      -0.4273567      -0.4765994      -0.3671580      -0.2268464
X_14     0.1715618       0.2132823       0.2577605       0.1768836       0.2316297       0.1974830       0.1286942       0.1562647
X_15    -0.5066662      -0.6625747      -0.3449860      -0.4198363      -0.5776442      -0.6699408      -0.5299194      -0.3348022
X_16     0.2082964       0.2585506       0.3216679       0.2120072       0.2509583       0.2396707       0.1202610       0.1724250
X_17     0.1714166       0.2201401       0.2846777       0.2447020       0.1084788       0.1943505      -0.2978895E-01   0.2138037
X_18     0.5066681       0.6625770       0.3449873       0.4198377       0.5776455       0.6699910       0.5290197       0.3348039
X_19     0.4617758       0.5309522       0.2656518       0.3638374       0.4273590       0.4766024       0.3871596       0.2268479
X_20     0.1470038       0.7226637E-01   0.4336375E-01   0.8742249E-01   0.2774912E-01  -0.2525463E-01  -0.1851781E-01  -0.1067964E-01
X_21     0.6943285       0.6509635       0.4811921       0.2662941       0.6031358       0.4990511       0.4971998       0.4324802
X_22     0.2201719E-01  -0.5466372E-02   0.2224674       0.6845814E-01   0.5984375      -0.4806109E-01   0.4015550       0.2541198
X_23     0.9117046       0.8017884       0.6634867       0.3806128      -0.1009850E-01   0.5536989      -0.4566940E-01   0.4833442E-01
X_24    -0.2385727      -0.2140442      -0.1603743      -0.1897873      -0.1147460      -0.1312916      -0.4039574E-01  -0.1684950
X_25     0.9980999       0.8910973       0.7316905       0.4457873      -0.3415111E-01   0.6244982      -0.5417298E-01  -0.1055268E-01
X_26     1.000000        0.8906841       0.6882940       0.4179102      -0.3068706E-01   0.6285862      -0.4249101E-01  -0.1306282E-01
X_27     0.8906841       1.000000        0.6380382       0.5792905       0.2927933E-01   0.9028665       0.2386003E-01   0.3267606E-01
X_28     0.6882940       0.6380382       1.000001        0.6252937      -0.6229956E-01   0.3989488      -0.1676782       0.2079019E-01
X_29     0.4179102       0.5792905       0.6252937       1.000000       -0.8787435E-02   0.4578466      -0.1784057       0.2443926
X_30    -0.3068706E-01   0.2927933E-01  -0.6229956E-01  -0.8787435E-02   1.000000        0.8728456E-01   0.6754959       0.6205069
X_31     0.6285862       0.9028665       0.3989488       0.4578466       0.8728456E-01   1.000000        0.1291215       0.1596082E-01
X_32    -0.4249101E-01   0.2386003E-01  -0.1676782      -0.1784057       0.6754959       0.1291215       1.000000        0.2123913
X_33    -0.1306282E-01   0.3267606E-01   0.2079019E-01   0.2443926       0.6205069       0.1596082E-01   0.2123913       1.000000
X_34     0.4860872E-01   0.1450582E-02   0.2251003       0.7378662E-01   0.5449154      -0.6161269E-01   0.3623104       0.2125029
X_35     0.4435728       0.6326291       0.2514482       0.3545317       0.6017353       0.6895741       0.5693176       0.3575687
X_36     0.4116380       0.5078289       0.1882765       0.3110257       0.4501029       0.4936658       0.4191048       0.2419259
X_37    -0.1570131      -0.2082819      -0.1375175      -0.2267920       0.3895626      -0.1899869       0.3669721       0.1579761
X_38     0.5208234       0.7719721       0.3509660       0.4586713       0.2638553       0.8645261       0.2936653       0.1042472
X_39     0.2047684       0.4228429       0.3195452       0.9392841       0.1667783E-01   0.3805327      -0.1428155       0.2877275
X_40     0.2372166       0.2224463       0.8261901E-01   0.1513010       0.3439495       0.1578265       0.2479531       0.3514436
X_41    -0.1870879      -0.1616225      -0.1160290      -0.1639674      -0.1162159      -0.8748719E-01  -0.4317056E-01  -0.1702979
X-42     0.8641571E-01   0.9617925E-01  -0.1437829      -0.3214717E-01   0.1246060E-01   0.1071582       0.2110054E-01   0.1764076
X-43     0.2337970       0.2995586       0.1386713      -0.1422618E-01   0.1337560       0.3473816       0.1747909       0.4154676E-01
X-44    -0.6326157E-01  -0.5340221E-01   0.2037252       0.1289405      -0.7126689E-01  -0.7146418E-01  -0.6333129E-01  -0.2676736
X-45    -0.4599456E-01  -0.1151574      -0.2888867E-03  -0.1202779      -0.7107437E-01  -0.1456470      -0.8821446E-01  -0.2400960E-01
X-46    -0.5146515E-02  -0.3783587E-01   0.2838597       0.5767628E-01   0.8770341E-01  -0.9039059E-01   0.9637032E-02  -0.1238275
X-47    -0.1665735      -0.2593153      -0.1451286      -0.2835524      -0.7144290E-01  -0.2641552      -0.6173394E-01  -0.5974319E-01
X-48    -0.1847013E-01  -0.6565702E-01   0.2372176       0.2543587E-02   0.7096684E-01  -0.1075545       0.3367039E-01  -0.1859508
X-49     0.2443823       0.2703575       0.1580873       0.4542351E-01   0.2140192       0.2673180       0.2494859      -0.3409774E-01
X-50     0.2884963       0.2398175       0.3230642       0.3390162      -0.1781713       0.9631711E-01  -0.2431629      -0.5679207E-01
X-61     0.2267250       0.2383037       0.3513134       0.4288582       0.2297403E-01   0.1313086      -0.6491470E-01   0.6025968E-01
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TOT CORR SECTION    5

ROW          X_34             X_35             X_36             X_37             X_38             X_39             X_40             X_41
 X_2    0.6555960E-02   0.2815528        0.2897823        0.2041711        0.1867546        0.9782833E-01    0.2606365E-01   -0.1074172E-01
 X_3    0.2708179      -0.6987131E-01   -0.6467193E-01    0.5795674E-01   -0.1198370        0.2013806        0.8338832E-02   -0.6655365E-01
 X_4   -0.3737510      -0.6946248E-01   -0.7523036E-01   -0.2348233        0.6983912E-01   -0.1029608       -0.5434539E-02   -0.8703917E-01
 X_5   -0.3877131E-01  -0.3680897E-01   -0.9564519E-01    0.5870248E-01   -0.8595884E-01   -0.2693918       -0.1469216E-01    0.6806570E-01
 X_6   -0.1459336      -0.2044694       -0.3064889       -0.2064694E-01   -0.2321488       -0.3661305       -0.1511814E-01    0.1223845
 X_7   -0.2469069      -0.3093016       -0.3796983       -0.2299659       -0.1037850        0.1434708       -0.1966609       -0.3424520
 X_8   -0.2229329      -0.2972327       -0.3552466       -0.2062802       -0.1265858        0.1261423       -0.1574283       -0.3783494
 X_9   -0.2140140      -0.2936516       -0.3741503       -0.2231306       -0.1078227        0.1356165       -0.1642157       -0.3492687
X_10   -0.4818267E-01  -0.4045368E-01    0.1778227        0.5241640E-01   -0.1148273       -0.1768311        0.5441998E-01    0.1968991
X_11   -0.3157466E-01  -0.2532604E-01   -0.1805621        0.5832704E-01   -0.8134246E-01   -0.3534493       -0.3612987E-01    0.1526322
X_12   -0.1074963      -0.2497997       -0.5398809       -0.3263212E-01   -0.1492595       -0.4347879       -0.2412911        0.1674191
X_13   -0.2685257      -0.7635671       -0.9651040       -0.1629705       -0.5291957       -0.3249497       -0.4927939        0.4673124E-01
X_14    0.3794543       0.2138080       -0.2624165E-01   -0.3127573E-01    0.2241769        0.1913831        0.9361643E-01   -0.2271594
X_15   -0.2846562      -0.9496095       -0.7775381       -0.1993635       -0.7300683       -0.3580936       -0.4820089        0.1201484
X_16    0.4487339       0.2451215        0.9828490E-01   -0.6145680E-01    0.2617466        0.1159184        0.1307790       -0.1675794
X_17    0.1075166       0.1731223        0.1425520       -0.5599724E-01    0.2289427        0.1719497        0.3230976E-01   -0.8809972E-01
X_18    0.2846560       0.9496159        0.7775403        0.1993644        0.7308701        0.3580950        0.4820090       -0.1201442
X_19    0.2685266       0.7635747        0.9651111        0.1629719        0.5291994        0.3249509        0.4927958       -0.4672924E-01
X_20    0.2310922       0.8785933E-01    0.6182970       -0.2549076E-01   -0.8028183E-02    0.8699912E-01    0.2412431        0.5115451E-01
X_21    0.3695460       0.6897489        0.5577505        0.1310042        0.4995880        0.1368190        0.4139963       -0.2398121
X_22    0.9579154       0.2789317        0.2601736        0.3333563        0.4969075E-01   -0.1466838E-01    0.1122834       -0.7911348E-01
X_23    0.4566260E-01   0.4097780        0.3602946       -0.1413397        0.4274397        0.1704004        0.2740919       -0.2598891
X_24   -0.9271711E-01  -0.1653011       -0.8264456E-01   -0.1739307       -0.4191045E-02   -0.1600074       -0.1304803        0.9964027
X_25    0.6479019E-01   0.4381106        0.4027541       -0.1592263        0.5009723        0.2195372        0.2298895       -0.1855447
X_26    0.4860972E-01   0.4435728        0.4116380       -0.1570131        0.5008234        0.2047684        0.2372166       -0.1873972
X_27    0.1450582E-02   0.6326291        0.5078265       -0.2082819        0.7719721        0.4228429        0.2224463       -0.1616225
X_28    0.2251003       0.2514482        0.1882765       -0.1375175        0.3509660        0.3195452        0.8261901E-01   -0.1169299
X_29    0.7378662E-01   0.3545317        0.3110257       -0.2267920        0.4986713        0.9392841        0.1513010       -0.1239674
X_30    0.5446154       0.6037353        0.4501029        0.3895626        0.2638553        0.1687783E-01    0.3439495       -0.1152159
X_31   -0.6181260E-01   0.6895741        0.4936658       -0.1899868        0.8645261        0.3805327        0.1578265       -0.1749719E-01
X_32    0.3623104       0.5663176        0.4191048        0.3609721        0.2986653       -0.1428155        0.2470531       -0.4319056E-01
X_33    0.2125029       0.3525687        0.2419250        0.1578761        0.1042472        0.2877275        0.3514436       -0.1702339
X_34    1.000000        0.2323322        0.2264191        0.2426246        0.2068256E-01   -0.9353641E-02    0.1695027       -0.8727616E-01
X_35    0.2323322       1.000001         0.8084378        0.2836956        0.7484976        0.3199862        0.4594378       -0.1294013
X_36    0.2264191       0.8084378        1.000000         0.2340403        0.5448805        0.2948062        0.4769798       -0.5380655E-01
X_37    0.2426246       0.2836956        0.2340403        1.000000        -0.2287798       -0.2149670       -0.1736197       -0.1747922
X_38    0.2068256E-01   0.7484976        0.5448805       -0.2287798        1.000000         0.4512801        0.9465772E-01    0.3056257E-01
X_39   -0.9353641E-02   0.3199862        0.2948062       -0.2149670        0.4512801        1.000000         0.1474029       -0.1481572
X_40    0.1695027       0.4594378        0.4769798       -0.1736197        0.9465772E-01    0.1474029        1.000000        -0.1141903
X_41   -0.8727616E-01  -0.1294013       -0.5380655E-01   -0.1747922        0.3056257E-01   -0.1481572       -0.1141903        1.000000
X-42   -0.4260632       0.1075344        0.9924448E-01    0.1617584E-02    0.9482646E-01    0.2424839E-01    0.4567917E-01   -0.1599459
X-43   -0.4485692E-01   0.2409042        0.2062893       -0.6359220E-01    0.3033116       -0.7822281E-01    0.5602110E-01   -0.2599466E-01
X-44    0.4495627      -0.4684161E-01   -0.7477097E-02   -0.2288798E-01   -0.4450065E-01    0.6691521E-01    0.4259355E-01    0.3288377E-01
X-45    0.9572428E-02  -0.9504330E-01   -0.1531072        0.2512635E-02    0.2246617E-01   -0.1459091       -0.2394534        0.3004949
X-46    0.7358595      -0.5316074E-01   -0.4934824E-01   -0.3963279E-01   -0.9872681E-01   -0.6314926E-02    0.1144727       -0.1138632E-01
X-47   -0.1921841E-01  -0.1248150       -0.1810792        0.7711607E-01   -0.7562834E-01   -0.2805561       -0.2225313        0.2536342
X-48    0.6903258      -0.7693726E-01   -0.5344379E-01   -0.2197822E-01   -0.1192493       -0.1013272        0.7635367E-01   -0.1309279E-01
X-49    0.1989734       0.3342736        0.1769614        0.8244550E-01    0.3172467       -0.1436611E-01    0.4780953E-01   -0.1646454
X-50    0.3877312E-01   0.3681071E-01    0.9564507E-01   -0.5670095E-01    0.8595705E-01    0.2693915        0.1469244E-01   -0.6876839E-01
X-51    0.1459351       0.2044776        0.3064920        0.2064510E-01    0.2321467        0.3661293        0.1512300E-01   -0.1223857
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TOT CORR SECTION   6

ROW        X-42            X-43            X-44            X-45            X-46            X-47            X-48            X-49
 X_2    0.2297348       0.3439179E-01  -0.3014880       0.2203643      -0.2283281       0.1875703      -0.2460437      -0.8809459E-01
 X_3   -0.1790110      -0.1179205       0.1578906       0.1032822       0.1971023      -0.1094345E-01   0.5612748E-01   0.5699287E-01
 X_4    0.1658017       0.8117127E-01  -0.1985109      -0.3286453      -0.2373361      -0.2642525      -0.1233147      -0.2709312E-02
 X_5   -0.9979135E-01   0.1163706E-01  -0.1007032      -0.1478206      -0.7184774E-01  -0.4647717E-02   0.1661891E-02   0.1363960
 X_6   -0.1195144E-01  -0.1393799      -0.1333045      -0.9136891E-01  -0.1009368       0.4917588E-01  -0.1476203E-01   0.5672291E-01
 X_7    0.9159192E-04  -0.1662459       0.7765677E-02  -0.1849310      -0.1010461      -0.1308094      -0.1049177      -0.3552988E-01
 X_8    0.3498712E-02  -0.1684272       0.7625473E-02  -0.2606766      -0.9921229E-01  -0.1502427      -0.9764846E-01  -0.4712269E-01
 X_9   -0.1249633E-01  -0.1664681       0.2373641E-01  -0.1901271      -0.8176321E-01  -0.1463454      -0.8423114E-01  -0.1823319E-01
X_10   -0.5493648E-01  -0.5485225E-02  -0.4302027E-01  -0.7355112E-01  -0.1264008      -0.3236177E-01  -0.8862448E-01  -0.1226593
X_11   -0.2436728       0.5596345E-01  -0.6535757E-01   0.8845150E-01   0.1216165       0.1902233       0.2186655      -0.8864349E-01
X_12   -0.1011248E-01   0.5930579E-01  -0.6964529E-01   0.2343189       0.3357909E-01   0.4048580       0.1180281      -0.2117266E-01
X_13   -0.6187818E-01  -0.2012754      -0.2870869E-01   0.1466677      -0.6167852E-02   0.2087166       0.1814298E-02  -0.1259988
X_14   -0.1773864       0.3143048       0.2132236       0.2742617E-01   0.3038664       0.2471504E-01   0.3545402       0.3276452
X_15   -0.6332111E-01  -0.2359088       0.2811956E-02   0.8944160E-01  -0.1559978E-01   0.1603895       0.1497798E-01  -0.2733401
X_16   -0.1168932       0.4297354       0.2576507      -0.1476661E-01   0.4238223      -0.5104292E-02   0.3849040       0.2978361
X_17    0.3382432       0.2201607       0.8337431E-02   0.9523306E-02  -0.9476197E-01  -0.1564521E-01  -0.1314225       0.1786625
X_18    0.6332344E-01   0.2359095      -0.2811664E-02  -0.8943999E-01   0.1569049E-01  -0.1603797      -0.1497773E-01   0.2730470
X_19    0.6187961E-01   0.2012767       0.2870885E-01  -0.1466666       0.8168485E-02  -0.2087161      -0.1814210E-02   0.1269035
X_20   -0.9243637E-01   0.1634465       0.1518524      -0.1659158       0.1447246      -0.1540576       0.1619246      -0.3069923E-01
X_21    0.7727265E-01   0.2863269      -0.8737866E-01  -0.1130629       0.4868246E-01  -0.1875166       0.3759594E-01   0.3299834
X_22   -0.3619791      -0.3367571E-01   0.4065596       0.4348367E-02   0.6312398      -0.1622743E-01   0.5924406       0.2523429
X_23    0.9981042E-01   0.2663195      -0.5460738E-01  -0.7876402E-01  -0.6570771E-02  -0.1722256      -0.1065699E-01   0.2547959
X_24   -0.1519740      -0.3244062E-01   0.1875816E-01   0.2937566      -0.1652752E-01   0.2661295      -0.1019236E-01  -0.1733483
X_25    0.6897336E-01   0.2314817      -0.4213502E-01  -0.4722418E-01   0.1927996E-01  -0.1688401       0.2787811E-02   0.2433381
X_26    0.8641571E-01   0.2337970      -0.6326157E-01  -0.4599456E-01  -0.5146515E-02  -0.1665735      -0.1847013E-01   0.2443873
X_27    0.9617925E-01   0.2995586      -0.5340221E-01  -0.1151574      -0.3783587E-01  -0.2593153      -0.6565702E-01   0.2723575
X_28   -0.1437829       0.1386713       0.2037252      -0.2688867E-03   0.2838597      -0.1451286       0.2372176       0.1589873
X_29   -0.3214717E-01  -0.1422618E-01   0.1289405      -0.1202779       0.9767628E-01  -0.2835524       0.2543567E-02   0.4547351E-01
X_30    0.3246060E-01   0.1337560      -0.7126689E-01  -0.7107437E-01   0.8770341E-01  -0.7144290E-01   0.7096084E-01   0.2149192
X_31    0.1071982       0.3473816      -0.7146418E-01  -0.1456470      -0.9039950E-01  -0.2641572      -0.1075545       0.2673183
X_32    0.2110054E-01   0.1747909      -0.6393129E-01  -0.8821446E-01   0.9637032E-02  -0.6173394E-01   0.3387039E-01   0.2499859
X_33    0.1764076       0.4154676E-01  -0.2676736      -0.2400960E-01  -0.1238775      -0.5904019E-01  -0.1850608      -0.3409294E-01
X_34   -0.4260632      -0.4485692E-01   0.4495627       0.9572428E-02   0.7358595      -0.1921841E-01   0.6903258       0.1989734
X_35    0.1075344       0.2409042      -0.4684161E-01  -0.9504133E-01  -0.5316074E-01  -0.1298153      -0.7643726E-01   0.3142736
X_36    0.9924448E-01   0.2063893      -0.7477097E-02  -0.1531072      -0.4934824E-01  -0.1810792      -0.5344379E-01   0.1763614
X_37    0.1617584E-02  -0.6359220E-01  -0.3298796E-01   0.2512635E-02  -0.3963279E-01   0.7711607E-01  -0.2197822E-01   0.8244559E-01
X_38    0.9482646E-01   0.3033116      -0.4450065E-01   0.2246617E-01  -0.9872681E-01  -0.7562834E-01  -0.1112493       0.3177467
X_39    0.2424839E-01  -0.7822281E-01   0.6691521E-01  -0.1459091      -0.6314926E-02  -0.2805561      -0.1013272      -0.1436611E-01
X_40    0.4567917E-01   0.5602110E-01   0.4258355E-01  -0.2394534       0.1144727      -0.2225313       0.7633367E-01   0.4789953E-01
X_41   -0.1590459      -0.2590466E-01   0.3288377E-01   0.3004948      -0.1138632E-01   0.2536342      -0.1337279E-01  -0.1646454
X-42    1.000001        0.1970340E-01  -0.5988000      -0.8697712E-01  -0.7047889      -0.4290326E-01  -0.6401314      -0.2745649E-01
X-43    0.1970340E-01   0.9999998      -0.1027408E-01  -0.7254249E-01  -0.4075564E-01  -0.3752097E-01  -0.5644109E-02   0.9139860E-01
X-44   -0.5988000      -0.1027408E-01   1.000000       -0.6477171E-01   0.6565574      -0.7070255E-01   0.5322789       0.1096116
X-45   -0.8697712E-01  -0.7254249E-01  -0.6477171E-01   1.000000        0.8790422E-02   0.8086694       0.1521074E-01  -0.2841168E-01
X-46   -0.7047889      -0.4075564E-01   0.6565574       0.8790422E-02   1.000000        0.1801948E-01   0.9273531       0.1026192
X-47   -0.4290326E-01  -0.3752097E-01  -0.7070255E-01   0.8086694       0.1801948E-01   1.000000        0.1041909       0.6808430E-01
X-48   -0.6401314      -0.5644109E-02   0.5322789       0.1521074E-01   0.9273531       0.1041909       1.000009        0.1466702
X-49   -0.2745649E-01   0.9139860E-01   0.1096116      -0.2841168E-01   0.1026192       0.6808430E-01   0.1466702       1.000000
X-50    0.9979206E-01  -0.1163862E-01   0.1007037       0.1478207       0.7184875E-01   0.4649214E-02  -0.1661767E-02  -0.1363972
X-51    0.1194803E-01   0.1393792       0.1330079       0.9136653E-01   0.1000411      -0.4917866E-01   0.1476394E-01  -0.5671722E-01
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TOT CORR SECTION    7

ROW           X-50            X-51
 X_2      0.5726286E-01   0.2861826
 X_3      0.2045043       0.1246886
 X_4     -0.2420432      -0.2423292
 X_5     -1.000000       -0.4090779
 X_6     -0.4090831      -1.000005
 X_7     -0.1401927E-01  -0.1722172
 X_8     -0.5029796E-01  -0.2001113
 X_9     -0.3714358E-01  -0.1979161
X_10     -0.2817239E-01   0.2088361E-01
X_11     -0.2513720      -0.1821250
X_12     -0.3546475      -0.3291315
X_13     -0.1164460      -0.3768544
X_14      0.7262552E-01   0.8545554E-01
X_15     -0.6350803E-01  -0.2931414
X_16      0.1141046       0.1444698
X_17      0.6352550E-01   0.1213981
X_18      0.6350666E-01   0.2931387
X_19      0.1164451       0.3768536
X_20      0.1758803       0.2829795
X_21      0.4234380E-01   0.1332209
X_22      0.7764995E-01   0.1664944
X_23      0.1974429       0.1684211
X_24     -0.9395868E-01  -0.1506232
X_25      0.2985791       0.2429295
X_26      0.2884953       0.2267250
X_27      0.2398175       0.2383037
X_28      0.3230642       0.3513134
X_29      0.3390162       0.4288582
X_30     -0.1781713       0.2297403E-01
X_31      0.9631711E-01   0.1313086
X_32     -0.2431629      -0.6491470E-01
X_33     -0.5679207E-01   0.6025968E-01
X_34      0.3877312E-01   0.1459351
X_35      0.3681071E-01   0.2044776
X_36      0.9564507E-01   0.3064920
X_37     -0.5870095E-01   0.2064510E-01
X_38      0.8595705E-01   0.2321467
X_39      0.2693915       0.3661293
X_40      0.1469244E-01   0.1512300E-01
X_41     -0.6806839E-01  -0.1223857
X-42      0.9979206E-01   0.1194803E-01
X-43     -0.1163862E-01   0.1393792
X-44      0.1067037       0.1330079
X-45      0.1478207       0.9136653E-01
X-46      0.7184875E-01   0.1000411
X-47      0.4649214E-02  -0.4917866E-01
X-48     -0.1661767E-02   0.1476394E-01
X-49     -0.1363972      -0.5671722E-01
X-50      1.000000        0.4090810
X-51      0.4090810       1.000000
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ROW        X-35            X-36            X-37            X-38            X-39            X-40            X-41            X-42
X-35   1.000001        0.8084378       0.2836956       0.7484976       0.3199862       0.4594378      -0.1294013       0.1075344
X-36   0.8084378       1.000000        0.2340403       0.5448805       0.2948962       0.4769798      -0.5380655E-01   0.9924448E-01
X-37   0.2836956       0.2340403       1.000000       -0.2287798      -0.2149670      -0.1736197      -0.1747922       0.1617584E-02
X-38   0.7484976       0.5448805      -0.2287798       1.000000        0.4512801       0.9465772E-01   0.3056257E-01   0.9482646E-01
X-39   0.3199862       0.2948962      -0.2145670       0.4512801       1.000000        0.1474029      -0.1481572       0.2424839E-01
X-40   0.4594378       0.4769798      -0.1736197       0.9465772E-01   0.1474029       1.000000       -0.1141903       0.4567917E-01
X-41  -0.1294013      -0.5380655E-01  -0.1747922       0.3056257E-01  -0.1481572      -0.1141903       1.000000       -0.1523459
X-42   0.1075344       0.9924448E-01   0.1617584E-02   0.9482646E-01   0.2424839E-01   0.4567917E-01  -0.1523459       1.000001
X-43   0.2409042       0.2063893      -0.6359220E-01   0.3033116      -0.7822281E-01   0.5602110E-01  -0.2593466E-01   0.1973340E-01
X-44  -0.4684161E-01  -0.7477097E-02  -0.3288756E-01  -0.4400065E-01   0.6691521E-01   0.4258355E-01   0.3288377E-01  -0.5988090
X-45  -0.9504330E-01  -0.1531072       0.2512635E-02   0.2246617E-01  -0.1459091      -0.2394534       0.3004948      -0.8697712E-01
X-46  -0.5316074E-01  -0.4934824E-01  -0.3963279E-01  -0.9872681E-01  -0.6314926E-02   0.1144727      -0.1138632E-01  -0.7047889
X-47  -0.1248150      -0.1810792       0.7711607E-01  -0.7502834E-01  -0.2805561      -0.2225313       0.2536342      -0.4270326E-01
X-48  -0.7693726E-01  -0.5344379E-01  -0.2197822E-01  -0.1192493      -0.1013272       0.7635367E-01  -0.1309279E-01  -0.6401314
X-49   0.3342736       0.1769614       0.8244550E-01   0.3172467      -0.1436011E-01   0.4780953E-01  -0.1646454      -0.2745549E-01
X-50   0.3681071E-01   0.9564507E-01  -0.5873091E-01   0.8595705E-01   0.2693915       0.1469244E-01  -0.6806839E-01   0.9979206E-01
X-51   0.2044776       0.3064920       0.2064510E-01   0.2321467       0.3061293       0.1512309E-01  -0.1223857       0.1194803E-01
X-52   0.4191809       0.3383644      -0.1464740E-01   0.4096938       0.1356760       0.1887808       0.6864985E-01  -0.8747634E-01
X-53  -0.1719702      -0.2023855       0.2827406      -0.3048908      -0.6375891E-01  -0.1590918      -0.5572134E-01  -0.1011072
X-54   0.2328770       0.2361673       0.1770772       0.1816055      -0.2202865E-01  -0.3374703E-01  -0.1911062      -0.3171574E-01
X-55   0.3231699       0.1749337       0.5342704E-01   0.3128259      -0.4722642E-01   0.6112517E-01  -0.1634218      -0.1106359E-01
X-56   0.3448306       0.2031304       0.9565264E-01   0.3227410      -0.2744372E-01   0.4371672E-01  -0.1832028      -0.2506610E-01
X-57  -0.1821687      -0.4116270E-01   0.4349918E-01  -0.1988806      -0.1571385E-01  -0.7885957E-01   0.8542246E-01  -0.7203302E-01
X-58  -0.3501343      -0.2504128      -0.1181973      -0.3136861      -0.9130472E-01  -0.5075891E-01   0.1330681       0.8674669E-01
X-59   0.2994417       0.1531494       0.5130062E-01   0.2909969      -0.5104355E-01   0.5274990E-01  -0.1616376       0.4872469E-02
X-60  -0.4810791E-01   0.5393789E-01  -0.1667779      -0.7824367E-01  -0.1738343E-01   0.2381188       0.1146705      -0.1881812
X-61  -0.9002696E-01  -0.2810480E-02   0.4755807E-01  -0.7280231E-01  -0.8494616E-01  -0.1196449      -0.5677753E-01  -0.6340539E-01
X-62   0.6992521       0.5060904      -0.2131694       0.8744674       0.5302747       0.1714235      -0.1370461       0.1026827
X-63  -0.6024960E-02   0.4110576E-01  -0.6402677E-01  -0.1644056E-01   0.2168450E-02   0.9985608E-01   0.1136010      -0.2439441E-01
X-64  -0.1972603E-01   0.3250088      -0.6969791E-01  -0.6621599E-01  -0.2424310E-01   0.1849914       0.3365049       0.6734413E-01
X-65   0.2897618       0.3159422      -0.1216367E-01   0.3590496       0.2440593       0.6099787E-02   0.1212472       0.1604031E-01
X-66  -0.2159182      -0.2032304      -0.6444103E-01  -0.1517575      -0.5764755E-01  -0.1116836       0.1133776      -0.1097429
X-67   0.2327908       0.1783967      -0.9228766E-01   0.3816370       0.2784652      -0.6364143E-01  -0.7233531E-01  -0.6108175E-02
X-68   0.3049976       0.2465662       0.5369115      -0.2244251E-01   0.1055164E-01   0.5472115E-01  -0.1553857      -0.2723308
X-69   0.1003234E-01   0.7222927E-01  -0.7083690E-01   0.1224757       0.3037885      -0.1000346       0.2729355       0.1595776
X-70   0.1789151       0.2369861      -0.4332699E-01   0.2184799      -0.6747782E-01   0.4880894E-01   0.1918441      -0.1170953
X-71  -0.1581212      -0.1867349       0.1991125      -0.2377431      -0.4840173E-01  -0.1902583      -0.2573790E-01  -0.4299662E-01
X-72   0.2199335       0.1538314       0.1362243       0.1786710       0.1759577      -0.2015256E-02   0.1045797       0.2259355
X-73  -0.2179573      -0.2790525      -0.1321200      -0.1620979       0.1712777      -0.1815530E-01  -0.2513719      -0.6428671E-01
X-74   0.3849062       0.3579354       0.1173165       0.2628613       0.1387049       0.2107076       0.1096944       0.3068612E-01
X-75  -0.3700445      -0.2905338      -0.1124037      -0.2738969      -0.1056716      -0.1662521      -0.1302471      -0.5285346E-01
X-76  -0.2056782      -0.3064202      -0.1851359      -0.1048957      -0.1942418      -0.5427462E-01  -0.2601390       0.3936299E-02
X-77   0.3558391       0.3120085       0.1038295       0.2422734       0.1377962       0.2021748      -0.4152713E-01  -0.1818832
X-78   0.1522843       0.1578574       0.3702827E-01   0.1311284       0.4035436E-01   0.4644954E-01   0.8508897E-01  -0.1524145
X-79  -0.5275977E-01  -0.2778216      -0.2294767       0.9452105E-01  -0.1179698E-01  -0.7775267E-02   0.4069723      -0.1778659
X-80  -0.1117325E-01  -0.1420677E-01   0.2116129E-01   0.9553343E-01   0.1555196      -0.2209387      -0.9942216E-01   0.7571591E-01
X-81   0.5649268E-01  -0.1974435      -0.1610092       0.1017014       0.1103610       0.1345690       0.1269562      -0.6360543E-01
X-82   0.3811724       0.3097725       0.8022600E-01   0.2882608       0.2719063       0.2052472       0.1588632      -0.2444603
X-83   0.2727853E-01  -0.2526202E-01  -0.5242057E-01   0.8566871E-02  -0.1586878       0.8789223E-01  -0.9426363E-02   0.3420281E-01
X-84   0.3400667       0.2024817       0.6299597E-01   0.3223835       0.6760567E-01   0.7402259E-01  -0.1610933       0.7962555E-02
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TOT CORR SECTION    2

ROW        X-43            X-44            X-45            X-46            X-47            X-48            X-49            X-50
X-35   0.2409042      -0.4684161E-01  -0.9504330E-01  -0.5316074E-01  -0.1248150      -0.7693726E-01   0.3342736       0.3681071E-01
X-36   0.2063893      -0.7477097E-02  -0.1531072      -0.4934824E-01  -0.1810792      -0.5344379E-01   0.1709614       0.9566507E-01
X-37  -0.6359220E-01  -0.3288798E-01   0.2512635E-02  -0.3963279E-01   0.7711607E-01  -0.2197822E-01   0.8244550E-01  -0.5070095E-01
X-38   0.3033116      -0.4450065E-01   0.2246617E-01  -0.9872681E-01  -0.7564834E-01  -0.1192493       0.3172467       0.8595706E-01
X-39  -0.7822281E-01   0.6691521E-01  -0.1459091      -0.6314926E-02  -0.2805561      -0.1013272      -0.1436611E-01   0.2693315
X-40   0.5602110E-01   0.4258355E-01  -0.2394514       0.1144727      -0.2225313       0.7635367E-01   0.4780953E-01   0.1450244E-01
X-41  -0.2590466E-01   0.3288377E-01   0.3004948      -0.1116632E-01   0.2530342      -0.1309277E-01  -0.1046454      -0.6678839E-01
X-42   0.1970340E-01  -0.5988000      -0.8697712E-01  -0.7047689      -0.4299326E-01  -0.6401314      -0.2745649E-01   0.9979206E-01
X-43   0.9999998      -0.1027408E-01  -0.7254249E-01  -0.4075564E-01  -0.3752097E-01  -0.5644109E-02   0.9139860E-01  -0.1163962E-01
X-44  -0.1027408E-01   1.000000       -0.6477171E-01   0.6565574      -0.7070255E-01   0.5322789       0.1096116       0.1007037
X-45  -0.7254249E-01  -0.6477171E-01   1.000000        0.8790422E-02   0.8046694       0.1521074E-01  -0.2841168E-01   0.1478207
X-46  -0.4075564E-01   0.6565574       0.8790422E-02   1.000000        0.1801948E-01   0.9273531       0.1026192       0.7184875E-01
X-47  -0.3752097E-01  -0.7070255E-01   0.8046694       0.1801948E-01   1.000000        0.1041909       0.6834430E-01   0.4642214E-02
X-48  -0.5644109E-02   0.5322789       0.1521074E-01   0.9273531       0.1041909       1.000000        0.1466702      -0.1661767E-02
X-49   0.9139860E-01   0.1096116      -0.2841168E-01   0.1026192       0.6834430E-01   0.1466702       1.000000       -0.1363372
X-50  -0.1163962E-01   0.1007037       0.1478207       0.7184875E-01   0.4642214E-02  -0.1661767E-02  -0.1363372       1.000000
X-51   0.1393792       0.1330079       0.9136653E-01   0.1000411      -0.4917866E-01   0.1476394E-01  -0.5671722E-01   0.4033919
X-52   0.7562023E-01   0.1666660       0.8700663E-01   0.6024427E-01  -0.1430796      -0.3446180E-01   0.1052788       0.3030642
X-53  -0.9680539E-01  -0.1052231E-01   0.2717222       0.1189588       0.3564708       0.1268830      -0.5173328E-01   0.6869147E-01
X-54   0.5708401E-01   0.9823525E-01  -0.8971304E-01   0.3982080E-01   0.8451127E-02   0.8169537E-01   0.7493562      -0.2564729
X-55   0.8401418E-01   0.6489227E-01   0.3578480E-01   0.1012059       0.1164319       0.1497751       0.8722839      -0.2307546E-01
X-56   0.8844393E-01   0.1095132      -0.2384751E-01   0.1010972       0.7748675E-01   0.1500448       0.9933454      -0.1311929
X-57  -0.1171195      -0.5836601E-01  -0.1329501      -0.8704454E-01  -0.1476126      -0.9704220E-01  -0.6455857      -0.1479152
X-58  -0.5560624E-01  -0.1719593       0.1790860      -0.9947509E-01   0.7128394E-01  -0.1184624      -0.7470449       0.1822186
X-59   0.9394437E-01   0.7649601E-01   0.5357445E-01   0.9496999E-01   0.1300713       0.1429102       0.7048821      -0.3325264E-01
X-60  -0.1587642E-01   0.1056289      -0.4974615E-01  -0.1220971      -0.1374184      -0.8274484E-01   0.7938755E-01  -0.1261066
X-61   0.7216728E-01   0.3324619E-01  -0.9029454E-01  -0.1179302      -0.1233277      -0.1144783      -0.1632571      -0.1042191
X-62   0.3042802      -0.5338003E-01  -0.1690105      -0.8401791E-01  -0.2930733      -0.1109874       0.2424902       0.1371443
X-63   0.3118700E-02   0.7891154E-01   0.9756494E-01   0.4076756E-01  -0.6973922E-01   0.4982940E-01  -0.1494123       0.1640117
X-64   0.3553338E-01   0.4496163E-01  -0.7665992E-01   0.3885582E-01  -0.5828213E-01   0.5413216E-01  -0.2412359       0.1226349
X-65   0.8451110E-01   0.7376324E-02  -0.2502754E-01  -0.2626327E-01  -0.6021824E-01   0.3259648E-01  -0.3816883E-01   0.1522194
X-66  -0.4300170E-01  -0.5830906E-02   0.2228863       0.2421353E-01   0.3782296       0.7153112E-01  -0.2116678E-01   0.1167471E-01
X-67   0.3439548E-01   0.2672639E-01   0.5170549E-01  -0.1388211E-01  -0.8109288E-01   0.1167032E-02   0.1244686       0.2109737
X-68  -0.2969518E-01   0.3269572       0.9665382E-01   0.5720056       0.9555370E-01   0.5456228       0.2635309       0.1253654
X-69  -0.6017696E-01  -0.1051671E-01   0.1416670      -0.1277779       0.1276659      -0.7196715E-01  -0.2112095       0.9296645E-01
X-70   0.1932860E-01   0.4388171E-01   0.9738100E-01  -0.2098908E-01  -0.3232408E-01  -0.3088602E-01   0.2758488E-01   0.1051104
X-71  -0.6760371E-01   0.1378105E-01   0.2651809       0.3904406E-01   0.3761933       0.1890797E-01  -0.1032249       0.1311798
X-72  -0.4810416E-01   0.2649068E-01   0.1964407E-01  -0.4535579E-01  -0.3797712E-01   0.5120294E-01   0.2527088E-01   0.1409110
X-73  -0.2638696       0.1297244E-01  -0.3721004E-01   0.6290072E-02  -0.5717677E-01  -0.1083697E-01  -0.8921194E-01   0.1274643
X-74   0.5921825E-01  -0.4130536E-01   0.6283466E-01  -0.3462291E-01   0.2401372E-01  -0.4793049E-01   0.7629836E-01  -0.3901781E-01
X-75  -0.7353222E-01   0.3345717E-01  -0.8439189E-01   0.2389703E-01  -0.5207591E-01   0.3248119E-01  -0.7176522E-01   0.3192909E-01
X-76   0.2377385E-01  -0.2484498E-01  -0.1561194      -0.2417909E-01  -0.4608670E-01   0.3192338E-01   0.2429023E-01  -0.3913651
X-77   0.5164053E-01  -0.8123034E-01   0.1024029E-01  -0.7778847E-01   0.1977404E-01  -0.5928268E-01   0.3596154E-01  -0.6368583E-01
X-78  -0.2544418E-01  -0.1274451E-01   0.7531369E-01   0.2607216E-01   0.2066869E-01  -0.6604291E-02   0.7441670E-01  -0.1083694
X-79   0.4069128E-03   0.8086610E-01   0.8433324E-03   0.9135640E-01  -0.6375986E-01   0.6744098E-02   0.2057461      -0.2792637E-01
X-80   0.3046971E-01   0.2509561E-01   0.2085261E-01  -0.2501279E-01  -0.2018350E-01  -0.9476513E-01   0.3755781E-01   0.2269192
X-81   0.3536920E-01   0.6563383E-01   0.1773608E-01   0.1097729      -0.1618692E-01   0.7025623E-01   0.2458109       0.1479177
X-82   0.2400145E-01  -0.7895964E-01   0.2056184E-01  -0.7213902E-01  -0.4216193E-01  -0.1199196      -0.6673032E-01  -0.1881569E-01
X-83   0.1723441      -0.9240378E-02  -0.2739726E-01  -0.3880816E-01   0.2352621E-01  -0.8925367E-02   0.1565763      -0.3966914
X-84   0.1252498       0.1519403      -0.1536151E-01   0.1129076       0.6401944E-01   0.1366504       0.9491491      -0.3040737E-01
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TOT CORR SECTION   3

ROW         X-51             X-52             X-53             X-54             X-55             X-56             X-57             X-58
X-35     0.2044776        0.4191809       -0.1719702        0.2328770        0.3221699        0.3446106       -0.1821687       -0.3591343
X-36     0.3064920        0.3183644       -0.2023855        0.2361673        0.1749337        0.2031304       -0.4116270E-01   -0.2590128
X-37     0.2064510E-01   -0.1464746E-01    0.2827406        0.1770772        0.5342704E-01    0.9565264E-01    0.4349918E-01   -0.1181973
X-38     0.2321467        0.4096938       -0.3048908        0.1616055        0.3128259        0.3227410       -0.1988806       -0.3136861
X-39     0.3661293        0.1356760       -0.6375851E-01   -0.2202895E-01   -0.4722642E-01   -0.2748372E-01   -0.1571395E-01   -0.9133472E-01
X-40     0.1512390E-01    0.1889808       -0.1520918       -0.3374708E-01    0.6112517E-01    0.4373672E-01   -0.7835957E-01   -0.5079891E-01
X-41    -0.1223857        0.6804985E-01   -0.5572134E-01   -0.1911062       -0.1604218       -0.1802028        0.8542246E-01    0.1339681
X-42     0.1194802E-01   -0.6740634E-01   -0.1611072       -0.3191574E-01   -0.1186359E-01   -0.2899610E-01   -0.7293302E-01    0.8678669E-01
X-43     0.1393792        0.7562023E-01   -0.9680539E-01    0.5708401E-01    0.8401418E-01    0.8844393E-01   -0.1171195       -0.5562624E-01
X-44     0.1330079        0.1666660       -0.1052731E-01    0.5823525E-01    0.8489227E-01    0.1095137       -0.5836601E-01   -0.1719593
X-45     0.9136653E-01    0.8700663E-01    0.2717222       -0.8571304E-01    0.3578480E-01   -0.2384751E-01   -0.1329501        0.1799383
X-46     0.1000411        0.6024427E-01    0.1189588        0.3982080E-01    0.1012059        0.1010772       -0.8204454E-01   -0.5947500E-01
X-47    -0.4917866E-01   -0.1430786        0.3564708        0.8451127E-02    0.1184319        0.7748675E-01   -0.1476126        0.7128394E-01
X-48     0.1476394E-01   -0.3446380E-01    0.1068830        0.8160537E-01    0.1497751        0.1500448       -0.9704220E-01   -0.1188824
X-49    -0.5671722E-01    0.1052788       -0.5173128E-01    0.7493562        0.8722839        0.9930454       -0.6456857       -0.7470449
X-50     0.4090810        0.3030642        0.6869145E-01   -0.2564729       -0.2397546E-01   -0.1311928       -0.1437152        0.1822186
X-51     1.000000         0.3292522        0.1775861        0.5549882E-02   -0.7412428E-01   -0.5796357E-01    0.3045519E-01   -0.9869143E-02
X-52     0.3292522        1.000001        -0.1682501       -0.8297979E-03    0.1523148        0.1143496       -0.9334338E-01   -0.3582460E-01
X-53     0.1775861       -0.1682501        1.000000        -0.1476810        0.4083270E-01   -0.4540291E-01   -0.1444027        0.2577830
X-54     0.5549882E-02   -0.8297979E-03   -0.1476810        1.000000         0.4145917        0.7335293       -0.9932125E-01   -0.7706148
X-55    -0.7412428E-01    0.1523148        0.4083270E-01    0.4145917        1.000000         0.9094813       -0.8341963       -0.4789865
X-56    -0.5796357E-01    0.1143496       -0.4540291E-01    0.7335293        0.9094813        1.000000        -0.6602435       -0.7299004
X-57     0.3045519E-01   -0.9334338E-01   -0.1444027       -0.9932125E-01   -0.8341963       -0.6602435        1.000000         0.9467506E-01
X-58    -0.9869143E-02   -0.3582460E-01    0.2577830       -0.7706148       -0.4789865       -0.7299004        0.9467506E-01    1.000000
X-59    -0.7109010E-01    0.1375142        0.5910817E-01    0.4792560        0.9854766        0.9298764       -0.8643491       -0.4583031
X-60     0.7651752E-01    0.1345564       -0.2019972        0.2562222E-01    0.1067371        0.8695747E-01   -0.6449419E-01   -0.1076555
X-61    -0.1084994        0.1257075       -0.2587827       -0.1286774       -0.7748532E-01   -0.1411460        0.5328757E-01    0.1549771
X-62     0.1869009        0.4951111       -0.2625768        0.9083968E-01    0.2695620        0.2508209       -0.1791894       -0.2537892
X-63     0.1798659       -0.7560643E-01   -0.9257449E-02   -0.1049484       -0.1569494       -0.1708041        0.1203402        0.7726634E-01
X-64     0.1381480        0.1747242E-01   -0.7972002E-01   -0.1874330       -0.2604276       -0.2576067        0.1573964        0.1097655
X-65     0.7957774E-01    0.7572192E-01   -0.7733256E-01   -0.6947343E-01   -0.1987235E-01   -0.3795227E-01    0.4517024E-01   -0.8816683E-01
X-66     0.6205629E-01   -0.7353759E-01    0.5418889       -0.7410628E-01    0.3178169E-01   -0.1628405E-01   -0.8089358E-01    0.1367964
X-67     0.4567173E-01    0.1992410       -0.7131648E-01    0.5371667E-02    0.1899812        0.1389658       -0.1614491       -0.8596174E-01
X-68     0.1614758        0.2317111E-01    0.1517957        0.2255219        0.2093136        0.2590101       -0.1761722       -0.2583667
X-69     0.1729228        0.1754672E-01   -0.6632996E-01   -0.2389126       -0.1874151       -0.2242943        0.6971633E-01    0.1477382
X-70     0.7677275E-01    0.4623121       -0.6197874E-01   -0.1280067        0.1533337        0.5058310E-01   -0.9865391E-01    0.6389305E-01
X-71     0.1842268       -0.1403799        0.7022917       -0.1369073       -0.5820411E-01   -0.1054915       -0.3367607E-01    0.1941646
X-72    -0.3758860E-01    0.2786522E-01   -0.1939754       -0.7630517E-04    0.2115863E-01    0.2242311E-01   -0.4578528E-01   -0.7851972E-01
X-73    -0.1613843       -0.2365755       -0.8317351E-01   -0.1366549       -0.1414044       -0.1205565        0.6403179E-01    0.8653519E-02
X-74     0.1177021        0.1429988       -0.5362760E-01    0.1015137        0.3456055E-01    0.7524997E-01    0.3168952E-01   -0.1644916
X-75    -0.1101364       -0.1461835        0.4460812E-01   -0.2681732E-01   -0.3081444E-01   -0.7014650E-01   -0.3287019E-01    0.1572845
X-76    -0.2723748       -0.2530593       -0.9229904E-01   -0.4014590E-01    0.1409119E-01    0.1701912E-01    0.5882960E-01   -0.6447589E-01
X-77     0.1308521        0.1576177        0.4441060E-01    0.6569797E-01   -0.2737913E-01    0.2555962E-01    0.7447004E-01   -0.1659004
X-78    -0.6543583E-01    0.1306207        0.3520934E-01    0.1251679        0.1339028E-01    0.7136285E-01    0.6452781E-01   -0.1981027
X-79    -0.2764047        0.7227629E-01    0.3788261E-01   -0.1782337        0.2776989        0.2156647       -0.4333705       -0.3279972E-01
X-80     0.3416387        0.2402599        0.8416113E-02    0.6828362E-01   -0.5303230E-02    0.3401872E-01    0.5632911E-01   -0.1445631
X-81    -0.2944769E-01    0.7030189E-01    0.1315003       -0.2365606        0.2299165        0.1788074       -0.4231828       -0.4196634E-01
X-82     0.5514370E-01    0.2086378        0.2369811E-01   -0.2954420E-01   -0.9679237E-01   -0.7408774E-01    0.1201079       -0.3353957E-01
X-83    -0.1433595       -0.8416426E-01   -0.1376350        0.1044909        0.1545092        0.1589382       -0.1555990       -0.1360856
X-84    -0.2239770E-02    0.6880057E-01   -0.6451094E-01    0.6511045        0.8696792        0.9450796       -0.7505079       -0.7128364
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TDT CORR SECTION    4

ROW        X-59            X-60            X-61            X-62            X-63            X-64            X-65            X-66
X-35    0.2994417      -0.4810791E-01  -0.9002686E-01   0.6902521      -0.6024960E-02  -0.1972603E-01   0.2097618      -0.2153182
X-36    0.1531494       0.5393789E-01  -0.2810480E-02   0.5060904       0.4110576E-01   0.3250384       0.3156422      -0.2033364
X-37    0.5130062E-01  -0.1667779       0.4755807E-01  -0.2131694      -0.6402677E-01  -0.8690779E-01  -0.1210367E-01  -0.6444103E-01
X-38    0.2909969      -0.7824367E-01  -0.7280211E-01   0.8744674      -0.1644056E-01  -0.6621599E-01   0.3599496      -0.1517575
X-39   -0.5104355E-01  -0.1708343E-01  -0.8494816E-01   0.5302747       0.2168450E-02  -0.2424312E-01   0.2449593      -0.5764755E-01
X-40    0.5274990E-01   0.2381188      -0.1196449       0.1714235       0.9985608E-01   0.1849914       0.6033787E-02  -0.1116936
X-41   -0.1616976       0.1146705      -0.5677753E-01  -0.1070461       0.1136010       0.3365949       0.1212472       0.1131776
X-42    0.4872460E-02  -0.1381812      -0.6343539E-01   0.1026827      -0.2439441E-01   0.6734413E-01   0.1604031E-01  -0.1007429
X-43    0.2394437E-01  -0.1887642E-01   0.7216728E-01   0.3042602       0.3118700E-02   0.3553338E-01   0.8451110E-01  -0.4300170E-01
X-44    0.7649601E-01   0.1056089       0.3224619E-01  -0.5338203E-01   0.7891154E-01   0.4496163E-01   0.7376324E-02  -0.5812906E-02
X-45    0.5357445E-01  -0.4974615E-01  -0.9029454E-01  -0.1690105       0.9756094E-01  -0.7665992E-01  -0.2592754E-01   0.2228863
X-46    0.9456999E-01  -0.1220971      -0.1179302      -0.8401793E-01   0.4026756E-01   0.3889582E-01  -0.2626327E-01   0.3421353E-01
X-47    0.1300713      -0.1374184      -0.1233277      -0.2930713      -0.6971922E-01  -0.5928213E-01  -0.6021324E-01   0.3792296
X-48    0.1429102      -0.8274484E-01  -0.1144783      -0.1169874       0.4598840E-01   0.5413216E-01   0.3259648E-01   0.7153112E-01
X-49    0.9948821       0.7938755E-01  -0.1632571       0.2424902      -0.1494123      -0.2412359      -0.3810083E-01  -0.2116678E-01
X-50   -0.3325264E-01  -0.1261066      -0.1042191       0.1371443       0.1640117       0.1226349       0.1523184       0.1167471E-01
X-51   -0.7109010E-01   0.7651752E-01  -0.1084994       0.1808009       0.1798669       0.1361480       0.7757774E-01   0.6205629E-01
X-52    0.1375142       0.1345564       0.1257075       0.4951111      -0.7530643E-01   0.1747242E-01   0.7572192E-01  -0.7353759E-01
X-53    0.5910817E-01  -0.2019972      -0.2587827      -0.2625768      -0.9257449E-02  -0.7972002E-01  -0.7733256E-01   0.5410992
X-54    0.4792560       0.2562222E-01  -0.1286774       0.9083968E-01  -0.1049484      -0.1674300      -0.8947343E-01  -0.7410628E-01
X-55    0.7854786       0.1067371      -0.7748532E-01   0.2635620      -0.1968494      -0.2604976      -0.1987235E-01   0.3178169E-01
X-56    0.9258764       0.8865747E-01  -0.1411460       0.2508209      -0.1708241      -0.2576767      -0.3795220E-01  -0.1628405E-01
X-57   -0.8643491      -0.6449419E-01   0.5328757E-01  -0.1793894       0.1203402       0.1579964       0.4557024E-01  -0.6009358E-01
X-58   -0.4583931      -0.1076555       0.1549771      -0.2537892       0.7722634E-01   0.1087655      -0.8746682E-01   0.1347964
X-59    1.000000        0.8604002E-01  -0.9415007E-01   0.2386150      -0.1864149      -0.2638528      -0.5457579E-01   0.4084236E-01
X-60    0.8604002E-01   1.000000        0.3448766      -0.2708175E-01   0.1381262       0.1078905       0.4956011E-02   0.1351830
X-61   -0.9415007E-01   0.3448766       1.000000       -0.6086297E-02  -0.1721182       0.4989334E-01   0.6932634E-01   0.2676243E-01
X-62    0.2386150      -0.2708175E-01  -0.6086297E-02   1.000000       -0.8887786E-01  -0.3195507E-01   0.3946811      -0.1319376
X-63   -0.1864149       0.1381262      -0.1721182      -0.8887786E-01   1.000000        0.2984223      -0.2342313E-01  -0.6734928E-01
X-64   -0.2638528       0.1078905       0.4989334E-01  -0.3195507E-01   0.2984223       0.9999998       0.4559235      -0.2998949E-01
X-65   -0.5457579E-01   0.4956011E-02   0.6932634E-01   0.3946811      -0.2342313E-01   0.4559235       1.000000       -0.7957005E-01
X-66    0.4084236E-01   0.1351830       0.2676243E-01  -0.1319376      -0.6734928E-01  -0.2998949E-01  -0.7957005E-01   1.000000
X-67    0.1684039       0.2221584E-01   0.1243217       0.4393771      -0.7971185E-01   0.9045348E-02   0.5598154      -0.5441781E-01
X-68    0.2178335      -0.2200083      -0.1948706      -0.5577836E-01   0.1449969      -0.1219193E-01  -0.5800509E-01  -0.3872140E-01
X-69   -0.1939868       0.1983610       0.7343853E-01   0.1286845       0.1268403       0.4622131       0.4106665       0.5015459E-01
X-70    0.1193919       0.2164533       0.1586769       0.2144462      -0.4855302E-01   0.7533169E-01   0.1416830E-01  -0.5407037E-01
X-71   -0.4475627E-01  -0.6672368E-01   0.8831132E-01  -0.1718763      -0.7308507E-01  -0.6418657E-01   0.1339875E-01   0.7615866
X-72    0.1378540E-01   0.8442223E-01   0.8507013E-01   0.2347646       0.1671494       0.7182533E-01   0.1243437      -0.7569647E-01
X-73   -0.1395868      -0.7947743E-02  -0.1716479      -0.9035510E-01  -0.7035968E-01  -0.3700359E-01  -0.8457452E-02  -0.1774867
X-74    0.2333375E-01  -0.3928284E-01   0.1712610E-01   0.2363450      -0.3314950       0.5393314E-01   0.1187008       0.3517660E-01
X-75   -0.2027464E-01   0.8055890E-01   0.3955044E-02  -0.2455987       0.3232191      -0.5015249E-01  -0.1595983      -0.4934455E-01
X-76   -0.5464047E-02  -0.7458437E-02  -0.3769576E-01  -0.1022531      -0.4342284E-01  -0.1331484      -0.2219906      -0.9529102E-01
X-77   -0.3817385E-01   0.2967729       0.1112028       0.2278563       0.9317446E-01   0.6693794E-01   0.1104420       0.3206594E-01
X-78    0.1493468E-02  -0.2656028E-01   0.5033055E-01   0.1027467       0.1907497E-01   0.4735225E-01   0.1465955E-01   0.3537443E-01
X-79    0.2969689       0.3335560E-01  -0.1066837       0.7790178E-01  -0.2339507E-01   0.7368639E-02  -0.1225236E-01   0.3074200E-01
X-80   -0.1725778E-01  -0.1770713      -0.8692068E-01   0.1307847       0.1252998E-01   0.1637478E-01   0.6904781E-01   0.1034362E-01
X-81    0.2516513      -0.2500734E-01  -0.1855772       0.1196556       0.6409919E-01   0.3678436E-01   0.6022445E-01   0.3435749E-01
X-82   -0.1088842       0.2790995       0.1605017       0.2854894       0.6770047E-01   0.3835464E-01   0.9967858E-01   0.1522565E-01
X-83    0.1555798       0.2211081       0.6647149E-01  -0.5942708E-01   0.2205174E-01  -0.7541341E-01  -0.8573210E-01  -0.7203341E-01
X-84    0.8968869       0.1024371      -0.1373857       0.2619548      -0.1341915      -0.1863379      -0.3440108E-02  -0.3138254E-01
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TOT CORR SECTION   5

ROW         X-67             X-68             X-69             X-70             X-71             X-72             X-73             X-74
X-35   0.2327908        0.3049976        0.1003234E-01    0.1789151       -0.1581212        0.2199335       -0.2179573        0.3849062
X-36   0.1782967        0.2465662        0.7222927E-01    0.2369861       -0.1087349        0.1530314       -0.2790525        0.3579354
X-37  -0.9228766E-01    0.5360115       -0.7083690E-01   -0.4312699E-01    0.1901125        0.1362241       -0.1321200        0.1173165
X-38   0.3816370       -0.2244251E-01    0.1224757        0.2184799       -0.2077431        0.1786710       -0.1620879        0.2628613
X-39   0.2784662        0.1055164E-01    0.3032845       -0.6747782E-01   -0.4840173E-01    0.1759577        0.1712777        0.1387049
X-40  -0.6364143E-01    0.5472115E-01   -0.1000386        0.4680694E-01   -0.1702583       -0.2015256E-02   -0.1815530E-01    0.2107096
X-41  -0.7203531E-01   -0.1553857        0.2729355        0.1918441       -0.2573790E-01    0.1043797       -0.2513719        0.1026944
X-42  -0.6198175E-02   -0.2723308        0.1505776       -0.1179953       -0.4299662E-01    0.2258055       -0.6429671E-01    0.3668612E-01
X-43   0.3439548E-01   -0.2969518E-01   -0.6017696E-01    0.1932860E-01   -0.6760371E-01   -0.4910436E-01   -0.2638686        0.5921925E-01
X-44   0.2672639E-01    0.3269572       -0.1051671E-01    0.4388371E-01    0.1378105E-01    0.2649268E-01    0.1297244E-01   -0.4130536E-01
X-45   0.5170549E-01    0.9665382E-01    0.1418670        0.9718100E-01    0.2651609        0.1964407E-01   -0.3721004E-01    0.6283408E-01
X-46  -0.1388211E-01    0.5720056       -0.1277779       -0.2098908E-01    0.3904406E-01   -0.4503579E-01    0.6829072E-02   -0.1468221E-01
X-47  -0.8108288E-01    0.9555370E-01    0.1276659       -0.2232408E-01    0.3761933       -0.3790712E-01   -0.5717677E-01    0.2401372E-01
X-48   0.1167032E-02    0.5450228       -0.7198715E-01   -0.3068602E-01    0.1893797E-01    0.5120291E-01   -0.1083697E-01   -0.4703049E-01
X-49   0.1244686        0.2635009       -0.2112095        0.2758488E-01   -0.1032249        0.2527088E-01   -0.8921194E-01    0.7629836E-01
X-50   0.2129737        0.1253654        0.9095645E-01    0.1051404        0.1311798        0.1400110        0.1274643       -0.3301781E-01
X-51   0.4587173E-01    0.1614758        0.1728228        0.7677275E-01    0.1842268       -0.3758860E-01   -0.1651833        0.1177021
X-52   0.1992410        0.2317111E-01    0.1754672E-01    0.4623121       -0.1403799        0.2786522E-01   -0.2365755        0.1429048
X-53  -0.7131648E-01    0.1517957       -0.6612996E-01   -0.8197874E-01    0.7032917       -0.1939754       -0.8317351E-01   -0.5163760E-01
X-54   0.5371667E-02    0.2255219       -0.2389126       -0.1280067       -0.1389073       -0.7639317E-04   -0.1366549        0.1015117
X-55   0.1899819        0.2093136       -0.1874151        0.1533337       -0.5820411E-01    0.2115463E-01   -0.1414044        0.3456055E-01
X-56   0.1389658        0.2590101       -0.2242943        0.5058310E-01   -0.1054915        0.2242811E-01   -0.1205565        0.7524997E-01
X-57  -0.1614491       -0.1761722        0.6971633E-01   -0.5865391E-01   -0.3987607E-01   -0.4528520E-01    0.6803179E-01    0.3368952E-01
X-58  -0.8586174E-01   -0.2583667        0.1477382        0.6380385E-01    0.1941646       -0.7851970E-01    0.8653510E-02   -0.1644916
X-59   0.1684039        0.2178335       -0.1939868        0.1193919       -0.4475627E-01    0.1378540E-01   -0.1395868        0.2333375E-01
X-60   0.2221584E-01   -0.2200081        0.1983610        0.2164533       -0.6672366E-01    0.8442223E-01   -0.7947743E-02   -0.3929284E-01
X-61   0.1243217       -0.1948708        0.7143853E-01    0.1586789        0.8801132E-01    0.8507013E-01   -0.1716479        0.1712610E-01
X-62   0.4393771       -0.5977836E-01    0.1286845        0.2144462       -0.1718763        0.2347646       -0.7035510E-01    0.2363450
X-63  -0.7971185E-01    0.1449949        0.1268403       -0.4855352E-01   -0.7308507E-01    0.1673494       -0.7055968E-01   -0.3314950
X-64   0.9945348E-02   -0.1219193E-01    0.4622131        0.7533169E-01   -0.6418657E-01    0.7182511E-01   -0.3700059E-01    0.5393314E-01
X-65   0.5998154       -0.5800509E-01    0.4106665        0.1416830E-01    0.1300875E-01    0.3245417       -0.8457452E-02    0.1182099
X-66  -0.5441781E-01   -0.3892140E-01    0.9015459E-01   -0.5407037E-01    0.7615866       -0.7579647E-01   -0.1774867        0.3513660E-01
X-67   1.000000         0.2269090E-01    0.2039075        0.1351408        0.3491501E-01    0.1197277        0.1226707        0.9458226E-01
X-68   0.2269090E-01    1.000000        -0.8511883E-01    0.2216516E-02    0.7684833E-01    0.1405964       -0.4183272E-01    0.1021045
X-69   0.2039075       -0.8511883E-01    1.000001        -0.2790252E-01    0.6085437E-02    0.3526047        0.6841207E-01    0.1628476
X-70   0.1351408        0.2216516E-02   -0.2790252E-01    1.000001        -0.1009907       -0.5565367E-01   -0.1573228        0.9677023E-01
X-71   0.3491501E-01    0.7684833E-01    0.6385437E-02   -0.1008907        1.000000       -0.1111721       -0.1275538        0.3067159E-01
X-72   0.1197277        0.1405964        0.3526047       -0.5565367E-01   -0.1111721        1.000000       -0.9430838E-01    0.1323153
X-73   0.1226707       -0.4181272E-01    0.6841207E-01   -0.1573228       -0.1275538       -0.9410834E-01    1.000001       -0.2475578E-01
X-74   0.9458226E-01    0.1021045        0.1628476        0.9677023E-01    0.3067159E-01    0.1323153       -0.2475578E-01    1.000000
X-75  -0.1093695       -0.1038544       -0.1819192       -0.9807473E-01   -0.4213645E-01   -0.1567394        0.2407430E-01   -0.9870965
X-76  -0.2003053       -0.1650915       -0.1803294       -0.1975029       -0.1417570       -0.2131096        0.2336817       -0.2659727
X-77   0.9367653E-01    0.9038335E-01    0.1565594        0.1022539        0.5374372E-01    0.8566004E-01    0.4219699E-01    0.1699126
X-78   0.4719581E-02    0.9506166E-01   -0.4241977        0.1033946        0.4399052E-01   -0.8568540E-02    0.4697971E-02    0.1863918E-01
X-79  -0.3925409E-02   -0.7179093E-01    0.3862903E-03    0.1401340       -0.2997120E-01   -0.5669188E-01    0.1015848        0.9949885E-02
X-80   0.4032492E-01    0.2815721E-01    0.5536181E-01    0.7342970E-01    0.2091359E-01    0.5247248E-01   -0.6705636E-01   -0.1511591E-01
X-81   0.6259060E-01    0.4967066E-01    0.7855636E-01    0.6460758E-01    0.6666440E-01    0.4705079E-01    0.2744682       -0.3571188E-01
X-82   0.8113271E-01    0.4600488E-01    0.1493712        0.1326089        0.2757021E-01    0.5774067E-01    0.5275284E-01    0.2149371
X-83  -0.6284237E-01   -0.6365275E-01    0.1622545E-01   -0.4705835E-02   -0.1252725       -0.8696109E-01   -0.2349312        0.8291811E-01
X-84   0.1514279        0.3011009       -0.1600233       -0.7380335E-02   -0.9008704E-01    0.7850017E-01   -0.7344484E-01    0.8019131E-01
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TOT CORR SECTION    6

ROW       X-75            X-76            X-77            X-78            X-79            X-80            X-81            X-82
X-35  -0.3700445      -0.2096782       0.3558391       0.1522843      -0.5275977E-01  -0.1117325E-01   0.5649268E-01   0.3811724
X-36  -0.2905338      -0.3064202       0.3120685       0.1578574      -0.2778216      -0.1420677E-01  -0.1974435       0.3097725
X-37  -0.1124037      -0.1851359       0.1034295       0.3702827E-01  -0.2294767       0.2116129E-01  -0.1610092       0.8022699E-01
X-38  -0.2738969      -0.1048957       0.2422734       0.1311284       0.9452105E-01   0.9553343E-01   0.1017014       0.2889609
X-39  -0.1056716      -0.1942418       0.1377962       0.4015436E-01  -0.1177698E-01   0.1555196       0.1103610       0.2719963
X-40  -0.1662521      -0.5427462E-01   0.2021748       0.4644954E-01  -0.7775269E-02  -0.2209087       0.1345690       0.2057472
X-41  -0.1302471      -0.2601390      -0.4352713E-01   0.8508897E-01   0.4689723      -0.9742216E-01   0.1269962       0.1588632
X-42  -0.5285346E-01   0.3586999E-02  -0.1818832      -0.1524145      -0.1778659       0.7571501E-01  -0.6360543E-01  -0.2444603
X-43  -0.7353222E-01   0.2377385E-01   0.5164053E-01  -0.2544418E-01   0.4609128E-03   0.3046371E-01   0.3536920E-01   0.2402145E-01
X-44   0.3345717E-01  -0.2484498E-01  -0.8123034E-01  -0.1274451E-01   0.8086610E-01   0.2505561E-01   0.6563193E-01  -0.7625764E-01
X-45  -0.8439189E-01  -0.1561194       0.1024029E-01   0.7531369E-01   0.8431324E-03   0.2085261E-01   0.1773608E-01   0.2056184E-01
X-46   0.2389703E-01  -0.2417904E-01  -0.7778847E-01   0.2607216E-01   0.9135640E-01  -0.2501279E-01   0.1317729      -0.7213702E-01
X-47  -0.5207591E-01  -0.4605670E-01   0.1977404E-01   0.2066869E-01  -0.6375988E-01  -0.2018350E-01  -0.1618692E-01  -0.4216193E-01
X-48   0.3248119E-01   0.3192938E-01  -0.5928268E-01  -0.6604291E-02   0.8744098E-02  -0.9476513E-01   0.7323623E-01  -0.1199196
X-49  -0.7376522E-01   0.2429023E-01   0.3596154E-01   0.7441670E-01   0.2657461       0.3755781E-01   0.2433109      -0.6673932E-01
X-50   0.3159299E-01  -0.3013651      -0.6363589E-01  -0.1083604      -0.2782637E-01   0.2269192       0.1474177      -0.1881669E-01
X-51  -0.1101354      -0.2723748       0.1308521      -0.6543583E-01  -0.2764047       0.3416397      -0.2744769E-01   0.5514372E-01
X-52  -0.1461835      -0.2530593       0.1576177       0.1306207       0.7227629E-01   0.2402599       0.7033189E-01   0.2096379
X-53   0.4460812E-01  -0.9202904E-01   0.4441060E-01   0.3520934E-01   0.3788261E-01   0.8416113E-02   0.1315903       0.2369811E-01
X-54  -0.8681732E-01  -0.4014690E-01   0.6569797E-01   0.1251679      -0.1782337       0.6828362E-01  -0.2365836      -0.2954420E-01
X-55  -0.3081444E-01   0.1409119E-01  -0.2737913E-01   0.1339028E-01   0.2776989      -0.5703230E-02   0.2299165      -0.9872337E-01
X-56  -0.7014650E-01   0.1701912E-01   0.2555962E-01   0.7130285E-01   0.2156647       0.3401872E-01   0.1788074      -0.7425774E-01
X-57  -0.3289019E-01   0.5082960E-01   0.7447004E-01   0.6452781E-01  -0.4330705       0.5602911E-01  -0.4231328       0.1221779
X-58   0.1562645      -0.6447589E-01  -0.1653004      -0.1583027      -0.3270972E-01  -0.1445631      -0.4106634E-01  -0.3351957E-01
X-59  -0.2027464E-01  -0.5464047E-02  -0.3812385E-01   0.1493460E-02   0.2969089      -0.1725778E-01   0.2516513      -0.1038942
X-60   0.8055890E-01  -0.7458437E-02   0.2967729      -0.2656028E-01   0.3335560E-01  -0.1770713      -0.2533734E-01   0.2719965
X-61   0.3955944E-02  -0.3762576E-01   0.1112028       0.5033055E-01  -0.1066837      -0.8692068E-01  -0.1855772       0.1625917
X-62  -0.2455987      -0.1022531       0.2278563       0.1027467       0.7793178E-01   0.1307847       0.1198556       0.2854904
X-63   0.3232171      -0.4342284E-01   0.9317446E-01   0.1907497E-01  -0.2319507E-01   0.1252999E-01   0.6409919E-01   0.6772347E-01
X-64  -0.5015240E-01  -0.1331484       0.6693758E-01   0.4735225E-01   0.7368619E-02   0.1637478E-01   0.3678430E-01   0.3835464E-01
X-65  -0.1595083      -0.2219906       0.1104420       0.1465955E-01  -0.1925236E-01   0.6904781E-01   0.8922445E-01   0.9967458E-01
X-66  -0.4934455E-01  -0.9525102E-01   0.3206594E-01   0.3537443E-01   0.3074200E-01   0.1034362E-01   0.3435749E-01   0.1522565E-01
X-67  -0.1293795      -0.2003053       0.9367663E-01   0.4719581E-02  -0.3923409E-02   0.4032492E-01   0.6259086E-01   0.8113271E-01
X-68  -0.1038544      -0.1650915       0.9938335E-01   0.9506166E-01  -0.7179099E-01   0.2815721E-01   0.4967066E-01   0.4664488E-01
X-69  -0.1819192      -0.1803294       0.1565594      -0.4241977       0.3862903E-03   0.5536181E-01   0.7655630E-01   0.1403712
X-70  -0.9807473E-01  -0.1975029       0.1022539       0.1033946       0.1401340       0.7142970E-01   0.6460798E-01   0.1329989
X-71  -0.4263645E-01  -0.1417570       0.5374372E-01   0.4399052E-01  -0.2997120E-01   0.2091059E-01   0.6656440E-01   0.2757921E-01
X-72  -0.1567394      -0.2131096       0.6566004E-01  -0.8566540E-02  -0.5689188E-01   0.5247248E-01   0.4705079E-01   0.5774367E-01
X-73   0.2469430E-01   0.2336817       0.4235695E-01   0.4687071E-02   0.1015848      -0.6705536E-01   0.2744682       0.5275784E-01
X-74  -0.9870965      -0.2559727       0.1699126       0.1863918E-01   0.9540385E-02  -0.1511591E-01  -0.3571188E-01   0.2149371
X-75   1.000000        0.2570766      -0.1207952       0.4787655E-02  -0.3280744E-01  -0.4270159E-02   0.1400138E-01  -0.1665089
X-76   0.2570766       1.000000       -0.1108237      -0.1644953       0.4462827E-01  -0.9104103E-01   0.1380485      -0.2576571
X-77  -0.1207952      -0.1108237       1.000000        0.2259743      -0.1355779       0.1274208E-01   0.2287600E-01   0.8248644
X-78   0.4787655E-02  -0.1644953       0.2259743       1.000000        0.4808670E-01  -0.1309125E-01  -0.3847294E-01   0.2566522
X-79  -0.3280744E-01   0.4462827E-01  -0.1355779       0.4808670E-01   1.000000       -0.8299983E-01   0.7797976       0.5339755E-01
X-80  -0.4270159E-02  -0.9104103E-01   0.1274208E-01  -0.1309125E-01  -0.8299983E-01   1.000000        0.1589902E-01   0.2398885E-01
X-81   0.1400138E-01   0.1380485       0.2287600E-01  -0.3847294E-01   0.7797976       0.1589902E-01   1.000000        0.6412446E-02
X-82  -0.1665089      -0.2576571       0.8248644       0.2566522       0.5339755E-01   0.2398885E-01   0.6412446E-02   1.000000
X-83  -0.8794165E-01   0.1838990E-01   0.2138926E-01  -0.1291090       0.2413882E-02  -0.1355961      -0.2202262      -0.2738714E-01
X-84  -0.7269615E-01  -0.2807506E-01   0.4814168E-01   0.8259720E-01   0.2679057       0.5035862E-01   0.2557016      -0.7064670E-01
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TGT  CORR  SECTION     7

ROW          X-83            X-84
X-35      0.2727853E-01   0.3400667
X-36     -0.2526202E-01   0.2024817
X-37     -0.5242057E-01   0.6295597E-01
X-38      0.8566871E-02   0.3223835
X-39     -0.1586878       0.6760567E-01
X-40      0.8789223E-01   0.7402259E-01
X-41     -0.9426363E-02  -0.1618933
X-42      0.3420281E-01   0.7962555E-02
X-43      0.1723441       0.1252498
X-44     -0.9240378E-02   0.1519403
X-45     -0.2739726E-01  -0.1536151E-01
X-46     -0.3880816E-01   0.1124076
X-47      0.2352621E-01   0.6401944E-01
X-48     -0.8925367E-02   0.1366504
X-49      0.1565763       0.9491491
X-50     -0.3066814      -0.3340737E-01
X-51     -0.1433595      -0.2239770E-02
X-52     -0.8416426E-01   0.6880057E-01
X-53     -0.1376350      -0.6451094E-01
X-54      0.1044909       0.6511045
X-55      0.1545092       0.6696792
X-56      0.1589282       0.9450796
X-57     -0.1555590      -0.7505079
X-58     -0.1360856      -0.7128364
X-59      0.1555798       0.8968869
X-60      0.2211081       0.1024371
X-61      0.6847149E-01  -0.1373857
X-62     -0.5942708E-01   0.2619548
X-63      0.2205174E-01  -0.1341915
X-64     -0.7541341E-01  -0.1863379
X-65     -0.8573210E-01  -0.1440108E-02
X-66     -0.7203341E-01  -0.3138254E-01
X-67     -0.6284237E-01   0.1514279
X-68     -0.6365275E-01   0.3011009
X-69      0.1622545E-01  -0.1600233
X-70     -0.4706835E-02  -0.7388335E-02
X-71     -0.1252725      -0.9608704E-01
X-72     -0.8696109E-01   0.7856017E-01
X-73     -0.2349312      -0.7344484E-01
X-74      0.8291811E-01   0.8019131E-01
X-75     -0.8794165E-01  -0.7269615E-01
X-76      0.1838990E-01  -0.2887566E-01
X-77      0.2138926E-01   0.4814168E-01
X-78     -0.1291090       0.6259720E-01
X-79      0.2413882E-02   0.2679057
X-80     -0.1355961       0.5035862E-01
X-81     -0.2262262       0.2557016
X-82     -0.2738714E-01  -0.7064670E-01
X-83      1.000000        0.2011809
X-84      0.2011809       1.000000
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APPENDIX H, PART I

VARIABLE STRUCTURE OF GROUP FAILED

Variable	Mean	Standard Deviation	Standard Error of Mean	Maximum	Minimum	F-Statistic
X-2	2.500000	0.9566892	0.1235080	5.000000	0.0	2.668944
X-3	2.645826	2.740095	0.3537447	9.000000	0.0	4.950998
X-4	6.836727	2.776093	0.3583919	10.00000	1.000000	20.69029
X-5	6.383333	5.831218	0.7528069	34.00000	2.000000	27.33253
X-6	39.18367	8.280439	1.068999	60.00000	25.00000	52.67641
X-9	0.1899633	0.1837232	0.2371855E-01	0.7867000	-0.2960000E-01	3.122393
X-10	2.303801	10.29112	1.328578	55.96989	-12.88090	1.949568
X-11	0.9263214	13.91727	1.796711	102.8907	-19.85579	0.1136909
X-12	-0.4725481	1.221793	0.1577328	0.4080999	-7.498400	15.64845
X-14	0.5817980	17.48567	2.257390	45.51208	-55.63170	0.6154320
X-15	0.2851364	0.2830713	0.3654435E-01	0.7873000	-1.081800	25.28076
X-17	0.3761680	16.62885	2.146774	21.07790	-119.8987	0.7022991
X-18	0.7148631	0.2830716	0.3654439E-01	2.081800	0.2126999	25.28110
X-19	0.8936597	0.4724157	0.6098860E-01	3.834200	0.2126999	27.09923
X-20	0.8385843E-01	2.918872	0.3768247	3.182300	-21.46999	0.1029068E-01
X-21	0.4080749	0.2546532	0.3287558E-01	0.9367000	0.0	4.69991
X-22	0.2224332E-01	0.3780302E-01	0.4880346E-02	0.2093000	0.0	18.05386
X-23	0.2278966	0.2149901	0.2775510E-01	0.9367000	0.0	4.639874
X-27	0.4701065	0.3892435	0.5025112E-01	1.570299	0.0	16.52359
X-28	0.2053833E-01	0.2829296E-01	0.3652605E-02	0.1372000	0.0	22.03366
X-29	0.5668831E-01	0.6480438E-01	0.8366209E-02	0.2818000	0.0	33.48315
X-30	0.165066	0.1706983	0.2203706E-01	0.8297000	0.0	0.6316518
X-33	0.3402665E-01	0.6410486E-01	0.8275900E-02	0.3349000	0.0	0.7226651
X-37	0.1386683	0.2112305	0.2726973E-01	1.070200	0.0	0.5831894
X-38	0.2920766	0.2249205	0.2903711E-01	0.7682000	0.0	9.772126
X-40	0.1449149	0.1301529	0.1680266E-01	0.5045000	0.0	3.589977
X-41	0.2737480	0.2587057	0.3339875E-01	1.027399	0.0	0.7568508E-01
X-42	1.322914	4.152411	0.5360740	32.18639	-3.222199	0.6493771
X-4[illegible]	2.[illegible]9875	44.66104	5.765715	187.1410	-267.4263	0.3629647

Variable	Mean	Standard Deviation	Standard Error of Mean	Maximum	Minimum	F-Statistic
X-44	0.1531917	3.542739	0.4573656	13.68660	-22.18379	0.1666017
X-45	0.2476949	0.8268071	0.1067403	3.699800	-0.9665999	2.770066
X-47	-0.2355668E-01	0.6619875	0.8546221E-01	3.258400	-1.468100	1.066381
X-48	1.383699	9.673825	1.248885	72.22879	-6.102900	1.011167
X-49	0.3844500E-01	0.2378771E-01	0.3070980E-02	0.6130000E-01	-0.4299998E-02	0.5817761
X-52	0.4409998E-01	0.1290876	0.1666514E-01	0.6296000	0.0	5.057554
X-53	0.6894493E-01	0.1374604	0.1774606E-01	0.7622000	0.0	1.016790
X-54	765.1680	36.74188	4.743356	839.2000	722.5000	0.2457964E-01
X-57	6.860497	1.133180	0.1462928	8.480000	5.820000	0.2297439E-01
X-58	7.262996	1.077483	0.1391025	8.480000	5.820000	1.974225
X-60	5.951521	0.9619000	0.1241807	6.897400	3.000000	0.2924239
X-61	3502.067	452.7112	58.44475	5343.000	2529.000	0.2805504
X-63	0.5972832E-01	0.7490754E-01	0.9670518E-02	0.3848000	0.9999999E-04	11.11113
X-64	23.63263	60.94411	7.867849	369.4722	0.2360000E-01	8.677245
X-65	0.1594649	0.4343243	0.5607102E-01	2.653299	0.9999999E-04	6.099426
X-66	0.3346463	0.5930182	0.7655829E-01	3.403999	0.7000000E-03	0.3702107E-01
X-67	0.6753993E-01	0.1022211	0.1319668E-01	0.6184000	0.0	13.19552
X-68	0.1421666E-01	0.2445292E-01	0.3156858E-02	0.1138000	0.0	10.44350
X-69	0.7233328E-01	0.8629304E-01	0.1114038E-01	0.3695000	0.0	6.700940
X-70	0.2631165E-01	0.7634193E-01	0.9855699E-02	0.4197000	0.0	5.048196
X-71	0.1973666E-01	0.4278952E-01	0.5524099E-02	0.2412000	0.0	0.3880569
X-72	0.6960660E-01	0.8276087E-01	0.1068438E-01	0.4055000	0.0	3.293394
X-73	2.084451	5.855290	0.7559147	35.86809	-15.79490	0.2256026E-01
X-75	4.062044	7.271001	0.9386821	54.20029	-0.4127000	5.031442
X-76	2.480680	5.842123	0.7542148	34.76849	-1.754700	25.69186
X-77	1.845468	6.024769	0.7777942	40.76649	-8.366300	4.021666
X-78	-1.913857	19.90100	2.569207	64.39659	-137.2993	5.740931
X-79	0.9710279	3.105337	0.4008973	8.115399	-15.25020	0.7917457
X-80	-0.2494267	3.695612	0.4771014	9.294399	-19.48340	1.949669
X-81	-1.569878	6.641584	0.8574248	19.48299	-21.46959	3.454923
X-82	3.300065	9.936050	1.282738	53.11130	-11.65420	4.365307
X-83	769987.4	1140052.	147180.1	6769102.	19431.00	5.333261

APPENDIX H, PART II

VARIABLE STRUCTURE OF GROUP NONFAILED

Variable	Mean	Standard Deviation	Standard Error of Mean	Maximum	Minimum
X-2	2.218183	0.9332087	0.1204767	4.000000	1.000000
X-3	1.462966	3.073743	0.3968185	10.00000	0.0
X-4	9.000000	2.421654	0.3126341	10.00000	0.0
X-5	13.36667	8.546912	1.103401	37.00000	3.000000
X-6	53.29628	12.58147	1.624261	79.00000	28.00000
X-9	0.2592833	0.2420467	0.3124809E-01	0.9719999	0.1600000E-02
X-10	0.4470013	0.4495794	0.5804044E-01	2.830199	-0.2858000
X-11	0.3201749	0.4037605	0.5212525E-01	0.9805000	-1.654599
X-12	0.1606316	0.2107965	0.2721370E-01	0.7940000	-0.5941000
X-14	2.412241	4.572142	0.5902609	20.25119	-8.952999
X-15	0.5553014	0.3051196	0.3939077E-01	1.000000	0.47100000E-01
X-17	2.220916	3.771632	0.4869156	20.25119	0.0
X-18	0.4446980	0.3051193	0.3939074E-01	0.9529000	0.0
X-19	0.4997449	0.3469633	0.4479276E-01	1.414200	0.0
X-20	0.1223800	0.3446766	0.4449746E-01	2.153500	0.0
X-21	0.3009297	0.2858382	0.3690155E-01	0.9030000	0.0
X-22	0.1381665E-02	0.4160158E-02	0.5370739E-03	0.2430000E-01	0.0
X-23	0.1481749	0.1896504	0.2448376E-01	0.8555000	0.0
X-27	0.2220799	0.2680846	0.3460957E-01	1.074699	0.0
X-28	0.2451665E-02	0.9502966E-02	0.1226828E-02	0.7189995E-01	0.0
X-29	0.7118329E-02	0.1426631E-01	0.1841773E-02	0.8409995E-01	0.0
X-30	0.1374616	0.2072207	0.2675208E-01	0.8599000	0.0
X-33	0.5175999E-01	0.1483204	0.1914807E-01	0.8462999	0.0
X-37	0.1102616	0.1959623	0.2529862E-01	0.7859000	0.0
X-38	0.1678783	0.2100479	0.2711707E-01	0.8548000	0.0
X-40	0.9812331E-01	0.1401887	0.1809828E-01	0.5125999	0.0
X-41	0.2654216	0.2718373	0.3509404E-01	0.9762000	0.0

Variable	Mean	Standard Deviation	Standard Error of Mean	Maximum	Minimum
X-42	0.8811030	0.8905859	0.1149741	5.043099	-1.137600
X-43	9.393896	76.11856	9.826863	518.0396	-173.7529
X-44	-0.8198154E-01	2.714315	0.3504166	2.913400	-19.40849
X-45	0.6443161E-01	0.2094023	0.2703371E-01	1.124700	-0.2661999
X-47	0.7115996E-01	0.2579497	0.3330115E-01	1.681199	-0.1774000
X-48	0.1219749	0.9370165	0.1209683	1.924100	-4.043099
X-49	0.3508165E-01	0.2450866E-01	0.3164054E-02	0.6130000E-01	-0.2140000E-01
X-52	0.4311662E-02	0.2415033E-01	0.3117794E-02	0.1853999	0.0
X-53	0.4620166E-01	0.1078271	0.1392041E-01	0.5858000	0.0
X-54	764.4829	35.41904	4.572577	839.2000	722.5000
X-57	6.895662	1.316834	0.1700026	11.28000	5.820000
X-58	7.539831	1.080339	0.1394712	8.480000	5.820000
X-60	5.856502	0.9477110	0.1223489	6.897400	3.301000
X-61	3545.667	451.4973	58.28804	5096.000	2726.000
X-63	0.2447165E-01	0.3318491E-01	0.4284151E-02	0.2068999	0.9999999E-04
X-64	0.4533198	0.9687832	0.1250693	4.885200	0.0
X-65	0.2053998E-01	0.3489218E-01	0.4504558E-02	0.1978000	0.0
X-66	0.4055865	2.792820	0.3605515	21.66679	0.0
X-67	0.1727999E-01	0.3219985E-01	0.4156981E-02	0.2139000	0.0
X-68	0.2891666E-02	0.1178594E-01	0.1521559E-02	0.8539999E-01	0.0
X-69	0.3668499E-01	0.6270570E-01	0.8095268E-02	0.2987000	0.0
X-70	0.3886666E-02	0.1220010E-01	0.1575026E-02	0.6200000E-01	0.0
X-71	0.1393333E-01	0.5810960E-01	0.7501915E-02	0.3904999	0.0
X-72	0.4227498E-01	0.8221710E-01	0.1061418E-01	0.4083000	0.0
X-73	2.233150	4.963823	0.6408268	18.49229	-13.86690
X-75	18.99142	51.03989	6.589220	354.2493	-4.220900
X-76	12.90677	14.82336	1.913688	87.59909	-3.773100
X-77	-5.647766	28.30887	3.654658	8.403700	-176.9331
X-78	-45.70370	140.1605	18.09464	0.7017000	-817.8048
X-79	-4.447937	47.07080	6.076814	33.05679	-353.5376
X-80	-2.846371	13.92460	1.797657	1.489300	-102.4324
X-81	-16.05777	60.00941	7.747182	31.53650	-400.4878
X-82	-3.356876	22.59145	2.916543	17.95580	-151.3875
X-83	1754666.	3099760.	400177.3	0.2012850E-08	5962.000

APPENDIX I

A PROFILE OF THE SELECTED VARIABLES FOR THE REDUCED SAMPLE

Variable	Discriminant Function Coefficients	Relative Power of Variables	Rank	Failed Group		Nonfailed Group		Univariate F ratio*
				Mean	Standard Deviation	Mean	Standard Deviation	
X- 5	0.00479	4.97%	10	6.00	5.12	13.84	9.02	23.02
X- 6	0.00580	9.35	5	38.63	8.82	53.56	13.65	34.48
X-12	0.25802	14.42	1	-0.25	0.57	0.17	0.22	22.15
X-45	-0.14338	12.97	2	0.40	0.98	0.07	0.23	5.29
X-58	0.07451	10.90	3	7.38	1.08	7.62	1.07	1.08
X-64	-0.00278	10.45	4	17.62	42.02	0.53	1.04	8.29
X-67	-0.95237	7.98	6	0.07	0.09	0.02	.03	12.85
X-73	-0.01012	6.52	8	2.10	3.58	1.67	5.43	0.17
X-75	0.00222	5.72	9	2.94	2.53	10.73	24.94	3.66
X-76	0.01117	7.62	7	0.55	2.28	6.21	6.31	27.70
X-81	-0.00085	4.61	11	-0.91	6.34	-14.15	52.16	2.42
X-82	-0.00411	4.51	12	3.25	11.80	0.16	2.89	3.21

Note: Most of the figures in the table have been rounded from 7 or more decimal places.

*$F_{1,28}(.01) = 7.64$; $F_{1,28}(.05) = 4.20$; $F_{1,28}(.10) = 2.89$.

APPENDIX J

EQUATION USED FOR CLASSIFICATIONS IN REDUCED SAMPLE

```
LINEAR AND QUADRATIC EQUATIONS IN TEST SPACE

  1.  LINEAR EQUATION U(FLD, NFLD) =

             VARIABLE X-5    VARIABLE X-6     VARIABLE X-12    VARIABLE X-45    VARIABLE X-58    VARIABLE X-64    VARIABLE X-67
             -0.81061E-01    -0.98117E-91      -4.3670           2.4267          -1.2611           0.46988E-01      16.119

             VARIABLE X-73   VARIABLE X-75    VARIABLE X-76    VARIABLE X-81    VARIABLE X-82
              0.17130        -0.37521E001     -0.18911          0.14428E-01      0.69513E-01

             CONSTANT1 = 13.494

             LOG OF P = 0.27444

  2.  QUADRATIC EQUATION U(FLD ,NFLD) =

             LINEAR TERMS    VARIABLE X-5     VARIABLE X-6     VARIABLE X-12    VARIABLE X-45    VARIABLE X-58    VARIABLE X-64
       X-5     2.5417         0.35702E-01
       X-6   '-1.0069        -0.80917E-02      0.8911E-02
       X-12   16.676          0.54493         -0.32392         -31.015
       X-45   -8.1305         0.33343E-01     -0.29861E-01      24.345          -26.072
       X-58    2.5921        -0.33756          0.66499E-01       0.93238          1.5798          -0.79097E-01
       X-64    0.92627        0.61041E-01     -0.38360E-01      -3.9799          -0.49030          0.35533          -1.2760
       X-67   -1.6691        -0.66996          1.7971           61.295          -39.642          -9.2045            20.566
       X-73   -0.22225       -0.58472E-01      0.62934E-03       0.70418         -0.16918          0.43235E-01       0.80749E-01
       X-75   -1.7057        -0.11290E-01     -0.69825E-02      -0.23004         -0.18066          0.12723          -0.18837E-01
       X-76   -2.8687        -0.57648E-01      0.26515E-01      -0.61608         -0.47618          0.23345          -0.92430E-01
       X-81    0.76148        0.18943E-01      0.24737E-02       0.86949E-01      0.38222E-01     -0.12518           0.12778E-01
       X-82    1.7154        -0.19440E-01      0.11503E-01      -0.25778E-02      0.14602         -0.23954           0.62304E-01
```

```
(SECTION 2)

           VARIABLE X-67   VARIABLE X-73   VARIABLE X-75   VARIABLE X-76   VARIABLE X-81   VARIABLE X-82
    X-67    -927.77
    X-73      -2.1561      0.70879E-01
    X-75   0.37371        -0.12367          0.24033
    X-76  -2.0529          0.39008E-01      0.10314         0.28388
    X-81   0.12927        -0.15201E-01     -0.33017E-01     0.42868E-01     0.35014E-01
    X-82  -1.1632         -0.24107E-01     -0.19630E-01    -0.85487E-02     0.10410E-01       -0.14050

           2 LOG OF P = 0.54887

           CONSTANT 1 = -8.7950

           LOG OF CONSTANT 2 = -3.0834

P = THE RATIO OF APRIORI PROBABILITIES OF GROUP J AND GROUP I AND/OR COSTS OF GROUP MEMBERSHIP

FOR QUADRATIC EQUATIONS CONSTANT 2 = DETERMINANT (DISPERSION MATRIX OF GROUP J)/DETERMINANT (DISPERSION MATRIX OF GROUP I)

QUADRATIC EQUATIONS U(I,J) ARE READ AS A HALF MATRIX OF COEFFICIENTS DEFINED BY THE PRODUCT OF THE ROW AND COLUMN TERMS,
PLUS LINEAR TERMS STORED IN THE FIRST COLUMN, PLUS CONSTANT 1.

QUADRATIC CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP 1 IF FOR ALL OTHER GROUPS J, THE EQUATION U(I,J) IS LESS THAN THE NATURAL LOG OF CONSTANT 2 MINUS 2
TIMES THE NATURAL LOG OF P

LINEAR EQUATIONS ARE READ AS A VECTOR OF COEFFICIENTS PLUS CONSTANT 1

LINEAR CLASSIFICATION RULES ARE AS FOLLOWS--
ASSIGN TO GROUP 1 IF FOR ALL OTHER GROUPS J, THE EQUATION U(I,J) IS GREATER THAN THE NATURAL LOG OF P
```

APPENDIX K

CLASSIFICATION OF THE REDUCED SAMPLE USING THE STANDARD METHOD

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	38 (100)	33 (86.84)	5 (13.16)	32 (84.21)	6 (15.79)
Nonfailed	50 (100)	5 (10.00)	45 (90.00)	2 (4.00)	48 (96.00)
Col. Total	88 (100)	38 (43.18)	50 (56.82)	34 (38.64)	54 (61.36)

Note: Figures in parentheses are percentages.

APPENDIX L

CLASSIFICATION OF THE REDUCED SAMPLE USING THE LACHENBRUCH METHOD

Actual Group	Total	Predicted Group By Quadratic Equation: Failed	Predicted Group By Quadratic Equation: Nonfailed	Predicted Group By Linear Equation: Failed	Predicted Group By Linear Equation: Nonfailed
Failed	38 (100)	31 (81.58)	7 (18.42)	31 (81.58)	7 (18.42)
Nonfailed	50 (100)	7 (14.00)	43 (86.00)	2 (4.00)	48 (96.00)
Col. Total	88 (100)	38 (43.18)	50 (56.82)	33 (37.50)	55 (62.50)

Note: Figures in parentheses are percentages.

APPENDIX M

CLASSIFICATION OF THE ORIGINAL SAMPLE USING THE "NORMALIZED" MODEL

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	28 (93.33)	2 (6.67)	30 (100.00)	0 (0.00)
Nonfailed	30 (100)	0 (0.00)	30 (100.00)	1 (3.33)	29 (96.67)
Col. Total	60 (100)	28 (46.67)	32 (53.33)	31 (51.67)	29 (48.33)

Note: Figures in parentheses are percentages.

APPENDIX N

CLASSIFICATION OF THE HOLDOUT SAMPLE USING THE "NORMALIZED" MODEL

Actual Group	Total	Predicted Group			
		By Quadratic Equation		By Linear Equation	
		Failed	Nonfailed	Failed	Nonfailed
Failed	30 (100)	24 (80.00)	6 (20.00)	25 (83.33)	5 (16.67)
Nonfailed	30 (100)	6 (20.00)	24 (80.00)	7 (23.33)	23 (76.67)
Col. Total	60 (100)	30 (50.00)	30 (50.00)	32 (53.33)	28 (46.67)

Note: Figures in parentheses are percentages.

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BIOGRAPHICAL SKETCH

Iraj Afkham was born on December 31, 1935, in Tehran, Iran, to Hossein and Batool Afkham. He was graduated from Tehran Nezam High School in June, 1955. In September of that year he enrolled in the Iranian Military Academy, receiving the degree of Bachelor of Science, and became an active second lieutenant in September, 1958. In March 1958 he married to the former Maryam Ghadery-Nejat.

While with the Iranian Army, he attended several educational courses and served in various commanding, staff, and teaching positions.

In January 1970 he graduated from Tehran University, with a master's degree in Public Administration. In December 1971 he received a master's degree in Business Administration from St. Louis University, St. Louis, Missouri.

At the present time he is a major of the Iranian Army and is assigned to the Iranian Armed Forces Supreme Commander's staff.

He and his wife have two daughters: Ladan, 13, and Mahnoosh, 7.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Henry R. Fogler, Chairman
Associate Professor of Management

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Robert C. Radcliffe, Co-Chairman
Assistant Professor of Finance

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Christopher B. Barry
Assistant Professor of Management

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Sanford V. Berg
Assistant Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Milton Z Kafoglis

Milton Z. Kafoglis
Professor of Economics

This dissertation was submitted to the Graduate Faculty of the Department of Management in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December, 1975

Harry H. Sisler
Dean, Graduate School

UNIVERSITY OF FLORIDA
3 1262 08552 9690